1.3 Propagation of Light

1.3.1 Light is Always ‘in a Hurry’...

In addition to understanding the exact nature of light, the understanding of light propagation has also attracted much attention and effort. A general principle, the principle of least time, or equivalently, the principle of minimum optical path, explains all phenomena relating to the propagation of light. This principle states that of all possible paths that light may follow from one point to another, it follows the path that corresponds to the least time. So, we see that light is always ‘in a hurry’!

Although credited to the French mathematician Pierre de Fermat, the Greek philosopher Hero of Alexandria had discovered that principle much earlier. In Hero’s version, the principle covered only the reflective part of light propagation. Fermat (in 1657) expanded the principle to include the refractive part of light propagation.

From this general principle, we derive the laws of reflection and refraction, as well as the principle of reversibility, which states that light will follow exactly the same path if its direction of travel is reversed. These principles and laws form the constitution of geometrical optics. The principle of reversibility is similar to the principle of least action, which applies in classical mechanics and is in full agreement with both the particle and the wave nature of light.

Figure 1-22: Examples of rectilinear propagation of light in free space. (left) Laser beams on an optical bench; (right) straight-line shadows falling on the living room in a winter day.
If we could record several screenshots of the electric field, we would see the \( \mathbf{E} \) vector vibrating on a plane perpendicular to the direction of propagation (denoted by \( \mathbf{r} \) in Figure 2-3), but with no specific orientation. We cannot locate a specific oscillation orientation because the orientation changes rapidly and randomly—again, we emphasize—along the plane that is perpendicular to the direction of propagation. Another rule, deriving from exactly the random nature of this orientation, is that at any given time and point, equal amounts of the vector magnitude \( E \) are projected along either the \(-x\) coordinate or the \(-y\) coordinate axis.

### 2.2 Linearly (Plane-) Polarized Light

If the oscillating electric field oscillates along only one direction (one axis) such that the orientation of the \( \mathbf{E} \) vector is fixed at all points in space and time, then the wave is called **linearly** (or **plane-**) polarized.

This fixed oscillation direction of the electric field is the **polarization axis**. The plane defined by the direction of propagation and the polarization axis is the **polarization plane**.

![Figure 2-4: Simple case of plane-polarized light. The wave propagates along the \(-z\) axis; the polarization axis is \(-y\), and the polarization plane is \(y-z\).](image)

The expression for the electric field associated with the wave depicted in Figure 2-4 is
Nature’s most spectacular display, the rainbow, has a strong linear polarization state. This is because the rainbow ray emerges from the droplet at an angle of \( \theta_t \approx 59° \)\(^{13}\), which is near Brewster’s angle of 53.08° for water.

Brewster’s angle (polarization)
- \( \tan^{-1}(n_2/n_1) \)
- valid from more dense to less dense optical media, and vice versa
- the angle of incidence for which reflection becomes linearly polarized
- angle of reflection \( \perp \) angle of refraction

Critical angle (total internal reflection)
- \( \sin^{-1}(n_2/n_1) \)
- valid from only more dense to less dense optical media
- the angle of incidence for which there is only internal and total reflection, i.e., no refraction
- angle of reflection = angle of incidence

Figure 2-47: Brewster’s angle versus TIR critical angle.

Figure 2-48: Photographs (left) without and (right) with a linear polarizer. The strong surface reflection is eliminated once a linear polarizer is used, indicating that the surface reflection light (for example, from the windshield and the hood) is linearly polarized. (Photos by Manutsawee Buapet www.bmanut.com used with permission.)

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13 Introduction to Optics § 3.5.2 Prismatic Atmospheric Phenomena.
3.3.2 Dispersion in Optical Glass

The dispersion curve depicts \( n(\lambda) \) versus \( \lambda \), i.e., the dependence of the refractive index on the wavelength. As shown in Figure 3-13, the specific material (flint glass) presents a refractive index distribution that ranges from 1.685 for the violet to 1.645 for the red. Note that with increasing wavelength, the value of the refractive index is decreasing. This is normal dispersion.

![Normal dispersion curve in a transparent material (flint glass) for the visible spectrum.](image)

The dimensionless Abbe number (named after the German physicist Ernst Karl Abbe), also known as constringence, is an expression of the medium’s dispersion and is defined as

\[
V = \frac{n_{V/d} - 1}{n_{B/R} - n_{B/C}}
\]

(3.48)

where \( n_{R/C} \) represents the red hydrogen spectral line (\( \lambda_R = 656.3 \text{ nm} \)), \( n_{V/d} \) represents the yellow sodium line (\( \lambda_V = 587.6 \text{ nm} \)), and \( n_{B/R} \) represents the blue hydrogen line (\( \lambda_B = 486.1 \text{ nm} \)).

If the refractive indices \( n_B \) and \( n_R \) are quite different, the Abbe number \( V \) has a relatively small value (such as \( V < 55 \)), indicating that the material is strongly dispersive (such as flint glass). If the indices \( n_B \) and \( n_R \) differ slightly, then \( V \) has a relatively large value (such as \( V > 55 \)), indicating that the material has a low dispersion.

<table>
<thead>
<tr>
<th>Lens Material</th>
<th>Refractive Index</th>
<th>Abbe Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>crown glass</td>
<td>1.523</td>
<td>58–60</td>
</tr>
<tr>
<td>CR-39 (plastic spectacle lens material)</td>
<td>1.498</td>
<td>58</td>
</tr>
<tr>
<td>Trivex® (aka Phoenix, NXT®, Trilogy®)</td>
<td>1.523</td>
<td>43–45</td>
</tr>
<tr>
<td>polycarbonate (plastic spectacle lens material)</td>
<td>1.586</td>
<td>30</td>
</tr>
<tr>
<td>dense flint glass</td>
<td>1.61</td>
<td>36.8</td>
</tr>
</tbody>
</table>
### 4.2.3 Transparent Plate: Thin-Film Interference

In many credit cards, there is an iridescent band that changes color depending on the observation angle. This is similar to the iridescent colors on oil slicks. These are examples of interference effects in thin, transparent slabs.

Figure 4-24 illustrates a glass plate with parallel surfaces, a fixed thickness $d$, and a refractive index $n$, surrounded by a medium with a refractive index $n_o$ (such as air). The plate is considered a **thin film** if the optical path inside it is smaller than the coherence length of the beam. It is considered transparent if the absorption inside the plate is minimal such that, after a pass through the glass, the exiting beam has an intensity comparable with that of the incident beam. Such a parallel-surface glass plate is illuminated with a thin monochromatic beam. We study the effect using the same method we applied in Young’s experiment. We seek:

- the interfering beams
- the geometrical factor that causes the optical path difference and therefore the phase difference.

![Figure 4-24: Interfering beams by reflection off a thin transparent plate.](image)

The interfering beams derive from the incident beam that is partially reflected off the upper surface (beam ❶) and partially refracted off this surface. The refracted beam is subsequently (partially) reflected off the inner surface (which is parallel to the upper surface) and exits the upper surface by refraction. This is beam ❷. The two surfaces, upper and lower, are termed **optically active surfaces**. Since it is the same beam that splits into two parts, this is interference with amplitude division.

Geometry indicates that the optical path difference between beams ❷ & ❶ (see Figure 4-25) is

$$\text{Optical Path Difference } (2 - 1): \quad n \cdot (AB + BC) - n_o(AD) \quad (4.52)$$
\[ \alpha = \frac{1}{2} k \alpha x_o = \frac{\alpha}{\lambda} x_o = \frac{\alpha}{\lambda} \sin \theta_z \]  

(5.23)

The diffraction field is then expressed, ignoring multiplicative constants, as

\[ E(x_o) = \text{sinc} \left( \frac{\alpha}{\pi} \right) = \frac{\sin(\alpha)}{\alpha} = \frac{\sin \left( \frac{\pi \alpha}{\lambda} \sin \theta_z \right)}{\frac{\pi \alpha}{\lambda} \sin \theta_z} \]  

(5.24)

The function \( \sin(\alpha)/\alpha \) has a maximum for \( \alpha = 0 \) and is zero for \( \alpha = \pm m \cdot \pi \) [rad], \( m = \pm 1, \pm 2, \ldots \).

\[ \text{Figure 5-23: Function } \frac{\sin(\alpha)}{\alpha}, \text{ single-slit diffraction field pattern.} \]

Between the zero crossings, there are phase changes (amplitude alternating from plus + to minus −). The diffraction intensity distribution is

\[ I(x_o) = E^2(x_o) = I_o \left( \frac{\sin(\alpha)}{\alpha} \right)^2 \]  

(5.25)

\[ \text{Figure 5-24: Diffraction intensity distribution pattern from a slit.} \]
### 5.5.2 Resolution Limit

Why is a diffraction-limited system important? Because if the optical system can form such a small point from an object point, then it also produces a sharp, crisp image from a real, extended object. It is about image quality.

We want to be able to distinguish the small details in such a crisp image. For example, say we are observing the details of a cell through a microscope. Optical resolution describes the ability of an imaging system to discern (resolve) detail. Two closely spaced but independent radiating points should be imaged to two distinct points. The concept of resolution applies to any imaging system, such as a telescope, a microscope, a photography camera, and, of course, the human eye.

The minimum separation (angular or spatial) between the closest distinguishable image points is the resolution limit. The lower the resolution limit, the better. The reciprocal of the resolution limit is the resolving power or resolving ability. The higher the resolving power, the better.

- **Resolution Limit**
  - ✔ The separation (expressed either angularly or spatially) between the closest distinguishable image points.
  - ✔ The smaller, the better.
  - ✔ Is expressed in angle (arcminute, milliradian) or length (millimeter) units.

- **Resolving Power (Ability)**
  - ✔ The reciprocal of the separation between the closest distinguishable points imaged through an optical instrument.
  - ✔ The higher, the better.
  - ✔ Is expressed in inverse angle (arcmin\(^{-1}\)) or inverse length (lines/millimeter, cycles/degree) units.

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**Figure 5-42:** (left) A single object imaged to an Airy disk, (center) two unresolvable spots, and (right) two marginally distinguishable spots, exhibiting the Rayleigh criterion.
Radiation emission involves processes of energy exchange between matter and light (as discussed in § 1.5.2) that result in light emission. Light emission is due to an electron (the ball in the above analogy) transition from an energy state \( E_2 \) to another state \( E_1 \) that is slightly lower. For the emission of a specific photon, the difference between the two atomic state energy levels corresponds to the photon frequency \( \nu_{12} \):

\[
h \cdot \nu_{12} = E_2 - E_1
\]

(6.16)

### 6.1.3.1 Spontaneous Emission

Consider two isolated energy states in an atom, states 1 and 2, between which an electric dipole transition is permitted. Ground level 1 has energy \( E_1 \), while the upper level 2 has energy \( E_2 \) (> \( E_1 \)). Electric dipole transitions are permitted between the two states.

We assume that the atom is in the excited state (one electron at level 2 and no electrons at level 1). This is an excited state, not the fundamental state, and is metastable: The system has a tendency to relax (decay) spontaneously when the electron returns to level 1 under no external influence. Thus, the system returns to the fundamental state, which is stationary since it corresponds to a lower energy. The life expectancy in the excited state depends, among other things, on the conditions of pressure and temperature, and does not exceed a few nanoseconds.

Upon relaxation, the energy difference \( E_2 - E_1 \) is released in the form of the emitted photon. This radiative process is termed **spontaneous emission**. The longer the mean lifetime of the electron in the excited state, the smaller the decay probability in the unit of time. The transition probability for spontaneous emission in the unit of time was expressed by Einstein as

\[
A_{21} \cdot dt
\]

(6.17)

where \( A_{21} \) is the coefficient of spontaneous emission. The quantity \( \tau_{\text{spont}} = 1/A_{21} \) has dimensions of time and is called the **transition lifetime** for the spontaneous emission. Coefficient \( A \) has dimensions of inverse time and is inversely proportional to the lifetime of the excited state. It relates to the mean lifetime and [per Eq. (6.12)] on the atomic structure, and is entirely independent of any field. Typical values for the lifetime are on the order of magnitude of \( 10^{-7} \) s.

![Figure 6-10: Spontaneous emission.](image)
able to perform all or most of their daily activities without spectacle glasses or contact lenses. Modern improvements to the procedure enable many patients to achieve 20/15 vision.

In LASIK, the flap was initially created with a mechanical blade, called a microkeratome. A fairly recent development is the femtosecond laser, which is a pulsed Nd:YAG laser in the near-infrared that can penetrate the cornea and focus inside it (the upper 100–120 μm), creating a lamellar cut. Thus, femtosecond-laser-assisted LASIK is the most advanced application.

Figure 6-64: (left) LASIK and (right) SMILE procedures for the correction of myopia.
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His doctorate research involved advanced optical signal processing and pattern recognition techniques (PhD, Tufts University, Massachusetts), and optical coherence tomography (Fellowship, Harvard University, Massachusetts). He then worked on research and development of optoelectronic devices in a number in research centers in the USA. He has authored more than 75 peer-reviewed research publications, 8 scholarly books on optics and optical imaging, and a large number of presentations at international conferences and meetings.

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