1.2.2 The Convex and Concave SSRI

The shape of the spherical surface comprising an SSRI can be convex [Figure 1-11 (left)] or concave [Figure 1-11 (right)]. By definition, a convex interface wraps around a medium of higher refractive index (center of curvature situated in the higher-index medium, e.g., glass), while a concave interface wraps around a medium of lower refractive index (center of curvature situated in the lower-index medium, e.g., air).

![Figure 1-11: (left) A convex SSRI. (right) A concave SSRI.](image)

1.2.3 Refraction by an SSRI

As in the case for any refracting surface, a ray incident on an SSRI is refracted. To properly draw the refracted ray from a spherical surface, we apply the law of refraction. We assume the simple case of a convex SSRI that separates air from glass and a ray that propagates parallel to the optical axis, striking the SSRI from the air side (Figure 1-12). The first step is to identify the center of curvature, which is the center of the hypothetical spherical surface that defines the SSRI. We then identify the normal to the surface at the point of incidence; this line draws along the spoke that connects the point of incidence to the center of curvature.

![Figure 1-12: Identification of the center of curvature and the normal to the surface at the point of incidence.](image)
Ray-Tracing Rules in an SSRI

1. The ray parallel to the optical axis refracts to the secondary focal point.
2. The ray originating from (or crossing through) the primary focal point refracts to become parallel to the optical axis.
3. The ray targeting the center of curvature refracts without any ray deviation (it crosses the center of curvature, which is the nodal point).

The topic of ray-tracing rules in an SSRI is further discussed in § 4.5.

Figure 1-27: Summary of ray tracing in a convex SSRI.

Figure 1-28: Summary of ray tracing in a concave SSRI.
2.4 LENS OPTICAL POWER

2.4.1 Lens-Maker’s Formula

A lens is a combination of two refracting surfaces with radii of curvature \( r_1 \) and \( r_2 \) that separate a medium with refractive index \( n_{\text{lens}} \) from a surrounding medium with index \( n_{\text{ext}} \).

![Diagram of a lens with radii of curvature and center of curvature](image)

To calculate the lens optical power, each surface that defines the lens may be considered as an independent surface. Then, we just add the individual optical powers \( F_1 \) and \( F_2 \) of each refracting surface, using Eq. (1.8), which expresses the SSRI optical power. The SSRI optical power is proportional to the difference of the refractive indices and is inversely proportional to the radius of curvature. Consider a lens made of glass with refractive index \( n_{\text{lens}} \) surrounded by a medium with \( n_{\text{ext}} \).

\[
F_1 = \frac{n_{\text{lens}} - n_{\text{ext}}}{r_1} \quad \text{and} \quad F_2 = \frac{n_{\text{ext}} - n_{\text{lens}}}{r_2} \quad (2.2)
\]

\[
F = F_1 + F_2 \quad \Rightarrow \quad F = \frac{n_{\text{lens}} - n_{\text{ext}}}{r_1} + \frac{n_{\text{ext}} - n_{\text{lens}}}{r_2} = \left( n_{\text{lens}} - n_{\text{ext}} \right) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \quad (2.3)
\]

If the lens is surrounded by air, then, simply, \( n_{\text{ext}} = 1.0 \). In addition, the optical power (reported in diopters, D) is simply the reciprocal of the lens focal length \( f \) (expressed in meters, m), which is the distance at which the lens focuses a collimated beam to a single point. Then, if the lens refractive index is denoted by \( n \), the lens power \( F \) in air is expressed as

\[
\text{Lens Power in Air:} \quad F = \frac{1}{f} = \left( n_{\text{lens}} - 1.0 \right) \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \quad (2.4)
\]

This relationship is known as the lens-maker’s formula, which is an approximate relationship, based on the following assumptions:
### 2.7 Lens Power Summary

**Focal Points**

A lens has two focal points. The *secondary*, or image-space focal point $F'$ is situated after a positive lens or before a negative lens. The *primary*, or object-space focal point $F$ is situated before a positive lens or after a negative lens.

**For a positive (converging) lens:**

The *secondary focal point* is the (real) image point if a collimated, parallel-to-the-optical-axis (plane wave) ray bundle enters the lens.

The *primary focal point* is the (real) object point that produces a collimated, parallel-to-the-optical-axis (plane wave) ray bundle leaving the lens.

![Figure 2-41: Secondary (left) and primary (right) focal points in a plus lens.](image)

**For a negative (diverging) lens:**

The *secondary focal point* is the (virtual) image point from which an originally collimated pencil of rays, when refracted by the lens, appears to originate as a diverging beam leaving the lens.

The *primary focal point* is the (virtual) object point to which a ray bundle appears to converge, prior to being refracted by the lens as a collimated beam (parallel plane wave).

![Figure 2-42: Secondary (left) and primary (right) focal points in a minus lens.](image)
3.1.1 Real and Virtual Object; Real and Virtual Image

The concept of object is directly tied to rays ‘leaving a point.’ Naturally, these rays (and, by association, the wavefronts) are diverging. An object is placed in front of (before) the optical element, and the rays reach the optical element (a positive lens, a negative lens, a mirror, or a single refracting interface) in a divergent configuration. This is a real object.

![Figure 3-2: A real object can be placed in front of a positive lens (left) or in front of a negative lens (right).](image)

A real object is the ‘common sense’ physical object, which is how ‘object’ has been described so far. We can conceptually extend the notion of an object to associate it with any light formation incident on the optical system. In optics, therefore, the object is associated with light incident on the optical system. However, light formation is not necessarily diverging. It is possible that the wavefront incident on the optical system is converging, although this does not occur naturally.\(^{13,14}\)

Assume, for a moment, that we remove the optical element (lens or a mirror). Light would converge to a point after (to the right of) that element—the location of the object, which, in essence, exists. The object is not physically formed due to the presence of a lens (or a mirror) in the way. This light formation incident on the optical element is a virtual object.

![Figure 3-3: A virtual object can be formed after a positive lens (left) or after a negative lens (right).](image)

\(^{13}\) Virtual objects are formed in multi-lens, or, in general, multi-element, imaging systems. A converging lens or a concave mirror will form a converging beam, forming a real image, which in turn can be incident on another optical element, forming a virtual image.

\(^{14}\) If the incident wavefront is flat, the incident vergence is zero. This object is located at optical infinity (see § 4.6.3).
Figure 4-47: Summary of the image formation configurations in lens imaging. [Left column: Positive lenses, top (1) to bottom (6)] 1, 2, and 3: Real object, real image. 4: (Real) object at the primary focal point, image at optical infinity. 5: Real object, virtual image. 6: Virtual object, real image. [Right column: Negative lenses, top (1) to bottom (6)] 1, 2, and 3: Virtual object, virtual image. 4: (Virtual) object at the primary focal point, image at optical infinity. 5: Virtual object, real image. 6: Real object, virtual image. (Compare to Figure 5-63.) Note: For simplicity, the rays shown correspond to two of the three construction rays (rules 1 and 3).
5.3.3 Ray Diagrams for Convex Mirrors

Alternatively, to find the image location and size, we can apply three simple ray diagram rules similarly to the way we applied lens ray diagram rules (presented in § 4.3). For a convex reflecting surface, the ray-tracing rules are as follows:

- A ray parallel to the principal optical axis (parallel ray) is reflected as if it originated from the focal point.
- A ray directed at (targeting) the focal point (focal ray) becomes parallel to the principal optical axis upon reflection.
- A ray directed at the center of curvature of the mirror (radial or nodal ray) is retro-reflected (simply reverses direction).

Figure 5-29: Ray-tracing diagrams for a convex mirror.
5.6 **Mirror Imaging Summary**

Mind the Sign!

In mirror imaging, all of the relationships are the same as their counterparts in lens imaging, and the algebraic signs follow the Cartesian sign convention (§ 3.2). The notation for object location is identical to that applied to lenses: An object location to the left of the mirror has a negative sign and to the right of the mirror has a positive sign.

![Figure 5-59: The Cartesian sign convention in mirrors for object location.](image)

The directional distances that apply to image space are associated with a reflected wave that has a reversed direction of propagation with respect to the initial, incident-to-the-mirror wave. Thus, image location, radius of curvature, and focal length values to the left of the mirror have a positive sign and to the right of the mirror have a negative sign.

![Figure 5-60: The Cartesian sign convention in mirrors for image location, radius of curvature, and focal length.](image)
Figure 5-63: Summary of image formation configurations. [Left column: Concave mirrors, top (1) to bottom (6)] 1, 2, and 3: Real object, real image. 4: (Real) object at the primary focal point, image at optical infinity. 5: Real object, virtual image. 6: Virtual object, real image. [Right column: Convex mirrors, top (1) to bottom (6)] 1, 2, and 3: Virtual object, virtual image. 4: (Virtual) object at the primary focal point, image at optical infinity. 5: Virtual object, real image. 6: Real object, virtual image. (Compare with Figure 4-47.)
6.2.2 Nodal Points

A thick lens has two **nodal points**. Their role is equivalent to that of the center of curvature in an SSRI (§ 1.2.6) or a mirror (§ 5.2.3), or the center of a thin lens. The undeviating ray (§ 4.3.1) crosses the center of a thin lens, maintaining its inclination with the optical axis. We note that there is no nodal plane; there are only nodal points. However, we use the notion of ‘nodal ray,’ which is a ray directed toward, or appearing to originate from, a nodal point.

The nodal points function as follows: A ray directed at the object-space nodal point $N$ emerges from the lens as if it originated from the image-space nodal point $N’$ without a change in the angle formed with the optical axis (parallel to its original direction).

![Figure 6-18: Nodal points in a thick lens and ray tilt preservation: Angle $\theta$, which expresses the ray tilt, is equal along both sides of the lens.](image)

The two focal points, the two principal points, and the two nodal points are the six **cardinal points**. These points are all situated on the optical axis.

*Cardinal Points*

- A thick lens or lens system has six (6) cardinal points:
- These are the two focal points, two principal points, and two nodal points.
- They are all situated on the optical axis.

If both sides of the lens are surrounded by the same medium ($n = n’$), the nodal points coincide with the corresponding principal points: $PN = P’N’ = 0$. If not ($n \neq n’$), the nodal points ($N$, $N’$) and the principal points ($P$, $P’$) are separated by

Principal-to-Nodal Point Displacement: $PN = P’N’ = f’ + f = \frac{n’}{F_e} - \frac{n}{F_e} = \frac{n’ - n}{F_e}$ (6.7)

*Note*: There is no direct formula for determining the nodal point locations; they are, essentially, referenced to their corresponding principal points. The good news is that Eq. (6.7) is not restricted to thick lenses but also applies to single refracting interfaces and reflecting surfaces.
The reciprocal of the back vertex power (in air) is the back focal length $BFL$ or $f_{BFL}$ measured from the back vertex point $V'$; likewise, the reciprocal of the front vertex power is the front focal length $FFL$ or $f_{FFL}$ measured from the front vertex point $V$.

**Front Surface Power $F_1$**

- Is the beam vergence that leaves the front (first) lens surface if a collimated beam (object at infinity) is incident on the lens.
- Is calculated using the SSRI power formula.
- Is referenced at the front (object-space) vertex point $V$.

**Equivalent Power $F_e$**

- Is the beam vergence that leaves the image-space principal plane $H'$ of a thick lens if a collimated beam (object at infinity) is incident on the lens.
- Is the sum of the front surface power $F_1$, the back surface power $F_2$, and the third term introduced by Gullstrand’s formula.
- Is referenced at the back (image-space) principal plane $H'$.

**Front Vertex Power $F_{FVP}$**

(also known as the neutralizing power)

- Is the beam vergence leaving the object-space front surface of a thick lens if a collimated beam is incident on the lens from the back side.
- Is the sum of the downstream-adjusted front surface power $F'_1$ and the back surface power $F_2$.
- Is referenced at the front (object-space) vertex point $V$.

**Back Vertex Power $F_{BVP}$**

(also known as the prescription power)

- Is the beam vergence leaving the image-space back surface of a thick lens if a collimated beam (object at infinity) is incident on the lens.
- Is the sum of the downstream-adjusted front surface power $F'_1$ and the back surface power $F_2$.
- Is referenced at the back (image-space) vertex point $V'$.

*Figure 6-79: Summary of power concepts and formulas in thick lenses.*
7.1.3.4 Locating the Pupils if the Aperture Stop is Unknown

If the aperture stop is unknown, to find the pupils (and the aperture stop), we follow these steps:

1. Form the images of any possible aperture stop (AS) in object space.
2. Identify the angle subtended from the on-axis object point to the edge of each AS image.
3. The image of the element with the smallest angle is the entrance pupil.
4. The element producing this image is the aperture stop.
5. The image of the aperture stop in image space is the exit pupil.

7.1.3.5 The Entrance Pupil in the Human Eye

The aperture stop of the eye is the anatomical iris. The entrance pupil is the object-space image of the iris, formed by the cornea, which is the ‘lens’ that is preceding it. For this imaging, the positive direction is from right to left. The object of the imaging is the iris opening, situated in the aqueous, which is the ad hoc object space with $n_{\text{aqueous}} = 1.336$.

To determine the entrance pupil of the human eye, we assume the following values: average corneal power $+42$ D; anatomical iris situated $3.6$ mm to the right of the cornea;\(^{31}\) eye filled with aqueous with refractive index $n_{\text{aqueous}} = 1.336$, and air with refractive index $n_{\text{air}} = 1.0$.

\(^{31}\) This is the anterior chamber depth. See Visual Optics § 2.4.2 Iris and Pupil.
We can compute the image-space aFoV considering the angular subtense of the exit port from the exit pupil. In the case where the object is at infinity, the image is formed at the focal plane of the lens. Now, the separation of the exit pupil from the exit port (distance $d_p$) equals the focal length of the lens $f$; therefore,

$$\text{Field of View (lens focused at infinity): } \quad \text{aFoV} = 2 \cdot \tan^{-1} \left( \frac{h}{f} \right)$$  \hspace{1cm} (7.4)

Figure 7-61: The aFoV when the lens is focused at infinity and the field stop is at the sensor (image) plane.

In many devices such as the photography camera, the sensor (field stop) has a fixed size, so the aFoV is inversely proportional to the focal length: The shorter the focal length, the larger the aFoV. This is why short-focal-length lenses (e.g., 35 mm or less) are considered to be wide field, while long-focal-length lenses (e.g., 125 mm or more) are considered to be narrow field.

Figure 7-62: Two photographs with a large and a small field of view, taken from the same spot (Molyvos, Lesvos Island, Greece) with different focal length lenses; (left) short-focal-length (35 mm) wide-angle lens with a large field of view and (right) long-focal-length (200 mm) telephoto lens with a small field of view.

When a lens is used as a collimating magnifier, the FoV that matters is the linear field of view, and specifically, just the linear extent of the viewable object. Consider a lens of focal length $f$ (power $F = 1/f$) with a semi-diameter $h$. The lens is held at a distance $d$. We seek the size of the linear field through this lens, which is simply the linear length of the object.

The object is situated at the primary focal point of the lens; therefore, the rays leave the lens collimated. If the lens diameter $2h$ is sufficiently larger than the eye’s pupil diameter, the
Principal Ray and Marginal Rays in a Three-Lens System

Example ☞: The system presented in Figure 7-110 comprises three lenses and one aperture stop (AS). The object and image locations are indicated, as well as the entrance pupil and exit pupil. Draw the marginal ray and the principal rays.

To draw the marginal ray and the principal ray, we follow the strategy outlined in § 7.2.2.1. The marginal ray originates at the on-axis object point and is initially directed toward the edge of the entrance pupil. It bends (refracts) at each lens, aiming at the on-axis image point of that lens upon refraction. On its way, it passes by the edge of the aperture stop and leaves the system by the edge of the exit pupil.

We note in Figure 7-111 that the marginal ray does not actually cross the edges of either the entrance pupil or the exit pupil; it is the extrapolation of the marginal ray that does so. This is because both the entrance pupil and the exit pupil are virtual images of the aperture stop.

Figure 7-110: Three-lens system. Note the two intermediate images and the entrance and exit pupil.

Figure 7-111: The marginal ray in the three-lens system.
8.3.3 Oblique/Radial Astigmatism

If the rays are tilted when they encounter the lens, or if the lens has a tilt with respect to the optical axis, in addition to the difference between the peripheral and paraxial rays (that causes coma), there is one more difference: The ray bundle may lie on a tilted plane, called the sagittal plane, or on a plane with no tilt with respect to the lens, called the tangential or meridional plane.

These two planes intersect the lens interfaces quite differently. As a result, the projected (perceived) lens optical thickness is different in each plane. Specifically, the ray pencil along the sagittal plane interacts with an increased lens thickness compared to the ray pencil along the meridional plane. Therefore, the lens optical power appears to be different along these planes. Thus, there exist two different focal lengths, depending on whether the rays are sagittal or meridional. This is oblique (or radial) astigmatism.
George Asimellis, PhD, serves as Associate Professor of Optics and Research Director at the Kentucky College of Optometry, Pikeville, Kentucky, which he joined in 2015 as Founding Faculty. He oversees development and coordination of the Geometric Optics and Vision Science courses and development of the Laser Surgical Procedures course.

In the past, he served as head of Research at LaserVision.gr Institute, Athens, Greece, and as faculty in: the Physics Department, Aristotle University, Greece; Medical School, Democritus University, Greece; and the Electrical Engineering Department, George Mason University, Virginia.

His doctorate research involved advanced optical signal processing and pattern recognition techniques (PhD, Tufts University, Massachusetts), and optical coherence tomography (Fellowship, Harvard University, Massachusetts). He then worked on research and development of optoelectronic devices in a number in research centers in the USA. He has authored more than 75 peer-reviewed research publications, 8 scholarly books on optics and optical imaging, and a large number of presentations at international conferences and meetings.

He is on the Editorial Board of eight peer-reviewed journals, including the Journal of Refractive Surgery, for which he serves as Associate Editor. He received the 2017 Emerging Vision Scientist Award by the National Alliance for Eye and Vision Research (NAEVR).

His research interests include optoelectronic devices, anterior-segment (corneal and epithelial) imaging, keratoconus screening, ocular optics, and ophthalmological lasers. His recent contributions involve publications in clinical in vivo epithelial imaging and corneal cross-linking interventions.