Electrons and Photons

2.1 Introduction

You will discover by measurement that all p-n diodes are sensitive to light, even if they are intended for some other application. A photodiode is a simple and inexpensive component that you will use to measure the particle behavior of light. This is a fundamental quantum-mechanical property of matter, and is the effect for which Albert Einstein was awarded the Nobel Prize in physics in 1921.

Photonic devices are used to convert photons to electrons and vice-versa. Photons and electrons are two of the basic quantum-mechanical particles. Like all quantum-mechanical particles, electrons and photons also behave like waves.

In this chapter, you will learn about the wave-like and particle-like aspects of the behavior of electrons and photons. Each electron that carries current in a semiconductor is spread out over many thousands of atoms; that is, it is delocalized. Trying to specify its position or its velocity is a hopeless task. Furthermore, the semiconductor is full of many absolutely identical electrons. They are all moving around at a frenetic pace. Clearly, a different approach is needed.

An important new idea in this chapter is to introduce a "road map" for electrons in a semiconductor. It tells you what states the electrons are allowed to occupy, just as a road map tells you where the roads are located that cars may travel on. The road map for electrons does not tell you where the electrons are or how fast they are moving, just as a roadmap for cars does not tell you where the cars are or how fast they are moving. This road map is called a *band structure*.

Position and velocity are not very useful ideas for describing either

electrons or photons. However, two fundamental physical laws always apply: conservation of energy and conservation of momentum. The behavior of electrons and photons can be tracked by their respective energies and momenta. The band structure is a particularly useful tool for this task.

2.2   The Fundamental Relationships

There are two simple principles that support almost all the science of photonic devices. One is the Boltzmann relationship and the other is Planck's equation relating the energy of a photon to the frequency of the light wave associated with the photon.

Ludwig Boltzmann

Boltzmann studied gases and the motion of molecules in gases. In a dense gas, Boltzmann said, the velocities of the molecules are statistically distributed about the average velocity $v_0 = 0$. Since the *Law of Large Numbers* in statistics says that all distributions tend toward a Gaussian or normal distribution, Boltzmann started from this point, too.

The probability of finding a particular velocity v_1 is given by a Gaussian distribution:

$$Pr(v = v_1) = A \cdot e^{\frac{-(v_1 - \overline{v_0})^2}{\langle v^2 \rangle}} \tag{2.1}$$

where $\overline{v_0}$ means the average velocity $= 0$, and $\langle v^2 \rangle$ means the average of the square of the velocity. Even though $\overline{v_0} = 0$, $\langle v^2 \rangle$ is definitely not equal to zero. This is the "spread" of the distribution.

Remember that:

$$E_{\text{kinetic}} = \tfrac{1}{2}mv^2$$

$$Pr(v = v_1) = A \cdot e^{\frac{-\frac{1}{2}m(v_1^2)}{\frac{1}{2}m\langle v^2 \rangle}}$$

$$\tfrac{1}{2}m\overline{\langle v^2 \rangle} = \text{spread in the energy} = \overline{E}$$

$$Pr(v = v_1) = A \cdot e^{-(E/\overline{E})} \tag{2.2}$$

From Brownian motion studies more than a century earlier, as well as mechanical equivalent of heat studies, energy is proportional to temperature. That is, $\overline{E} = \text{constant} \cdot T$ and

$$Pr(v = v_1) = Pr(E = E_1) = A \cdot e^{-(E/\text{constant} \cdot T)}$$
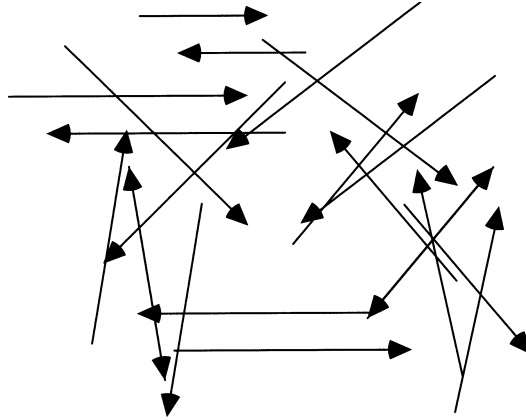
Figure 2.1. A schematic picture of a collection of atoms in a gas. The arrows give the magnitude and direction of the velocity of each atom. If the gas is contained in a bottle on your lab bench, then the average velocity of the atoms relative to you is 0. However, the average of the square of the velocity is a positive number.

So, what is this constant? Boltzmann's constant, of course!

$$Pr(E = E_1) = A \cdot e^{-(E/k_BT)}$$

$$k_BT \cong 0.026 \text{ eV @ 295 K = room temp} \tag{2.3}$$

If the total number of gas molecules in the bottle is N_T, the number of molecules having energy E_1 is given by the total number of molecules times the probability that a molecule has energy E_1:

$$n(E_1) = N_T Pr(E = E_1) = N_T \cdot e^{-(E_1/k_BT)} \tag{2.4}$$

The number of molecules at energy E_2 relative to those at energy E_1 is readily expressed:

$$\frac{n(E_2)}{n(E_1)} = e^{-(E_2-E_1)/k_BT} \tag{2.5}$$

The Boltzmann relation given in Eq. 2.5 is a fundamental tool that you use to determine how photonic devices operate. The Boltzmann relation can be applied to electrons as well as to molecules, provided that these electrons is are equilibrium. With suitable and simple modifications, it is possible to use this relationship under nonequilibrium conditions. The current–voltage expression for a p-n diode is exactly that adjustment. We will use this tool over and over throughout this book. Its importance cannot be overestimated.

2.3 Properties of Photons

a. According to Maxwell, light is an electromagnetic wave.
b. According to Michelson and Morley, light always travels at a constant speed, c.
c. speed of light = c = wavelength × frequency = $\lambda f \sim 3 \times 10^{10}$ cm/sec
d. visible light:

    400 nm $< \lambda <$ 700 nm (400 nm = blue, 700 nm = red)

  near infrared:

    700 nm $< \lambda <$ 2000 nm

There are many important applications in the visible and near-infrared regions of the spectrum, including the wavelengths that optimize optical fiber communications. The most important properties of optical fibers for communications are attenuation of the signal by absorption and distortion of the signal (noise).

High-performance optical fibers are made from glass. Attenuation is caused by fluctuations in the density of the glass on the atomic scale and from residual concentrations of water molecules. The water molecules absorb light near specific wavelengths. In between these wavelengths, windows of lower attenuation are formed at λ = 1300 nm and λ 1500 nm. A good picture of this situation is shown in Fig. 2.2 for state of the art optical fibers. The properties of several types of fibers, all of which are made by chemical vapor deposition, are shown. The properties of optical fibers are covered in more detail in Chapter 9.

Another important application for infrared wavelengths is night vision binoculars. These instruments are composed of detectors that image the infrared heat radiation from objects and convert this signal to a visible image so that the wearer can see in the dark.

Light beams behave like waves, and the wave properties of light are easy to observe:

- diffraction effects
- dispersion effects; for example, a rainbow
- interference effects
- wavelength
- frequency

Light beams also display effects associated with particles. These effects are not as apparent in everyday experience. In the laboratory, you will observe this behavior often.
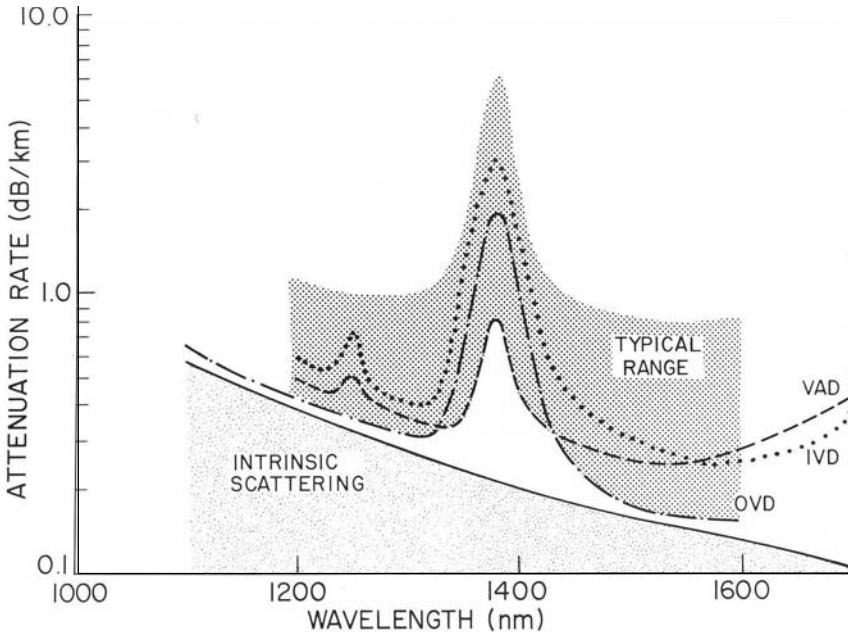
Figure 2.2. Optical fibers are made of glass and can be very transparent if the glass is pure. At 1500 nm, the loss is about 0.2 dB per kilometer. This means that a kilometer of optical fiber is about as transparent as an ordinary windowpane. Fibers are drawn like taffy from a preform. The properties of preforms made in three different ways are shown: vapor axial deposition, outside vapor deposition, and inside vapor deposition. The large loss peak at 1400 nm is the result of absorption by the first harmonic of residual OH molecules in the glass. Please see Chapter 9 for more details. (Adapted from D. Keck et al., *Proc. SPIE,* by permission.

Let us look at Planck's study of incandescent radiation.

Observation: when things get hot, they begin to glow. As they get hotter, (1) they glow more brightly and (2) the color of the glow changes. We can measure the color of the glow by the frequency of the light. So there seems to be a relationship between temperature and frequency (color).

Exercise 2.1

If you have an electric heating appliance, you can try the following experiment. After turning off the room lights, turn on the appliance and watch it as it heats up. Record your observations.

Note: Some people have sensitivity to infrared wavelengths beyond the range of normal vision. According to Edwin Land, inventor of the Polaroid camera, who studied this effect, the "color" associated with

this sensitivity is yellow. It appears just before the dark red glow of the heating element appears in the visible range as it warms up. In my classes, this effect is seen by about one out of thirty students. Sensitivity does not appear to depend on age or sex.

Planck's proposition was that temperature is proportional to frequency. But Boltzmann already knew that temperature is proportional to energy. Therefore, we conclude that color is proportional to energy. As the energy goes up, how does the frequency change?

Remembering that $\lambda f = c$, as the energy gets larger, does the wavelength increase or decrease? As the energy gets larger, does the frequency increase or decrease?

So, of the two things that characterize light, λ and f, which one is proportional to the energy? As the energy goes up, the wavelength gets shorter or smaller. However, the frequency has to increase because $\lambda f = c$. Thus, *energy is proportional to frequency:*

$$E = hf \tag{2.6}$$

h, of course, is Planck's constant.

Energy in a monochromatic beam of red light equal to $n \cdot h \cdot f$(red light), where n is the amplitude, or the number of vibrations, each one of which carries hf of energy:

$$energy = \sum_f hf \cdot n_f \qquad \text{over all frequencies}$$

where n_f is the number of photons distributed according to Bose–Einstein statistics:

$$n_f = \text{const} \cdot \left(\frac{1}{e^{hf/k_B T} - 1} \right) \tag{2.7}$$

When $hf > k_B T$, such as in the case of an incandescent body like a stove element, n_f is distributed to a good approximation by Boltzmann's law.

Some important results obtained so far are:

1. Boltzmann's law. For a group of electrons at equilbrium,

$$\frac{n(E_2)}{n(E_1)} = e^{-(E_2 - E_1)/k_B T}$$

2. Energy is proportional to frequency: $E = hf$, where h is Planck's constant, equal to 6.63×10^{-34} joule-sec.

Exercise 2.2

Take λ = 1000 nm = 1 μm = 10^{-4} cm. For a tungsten light bulb, this is the wavelength of peak intensity. What is the energy associated with this wavelength?

Procedure:

$$\lambda f = c, \quad \text{or} \quad f = c/\lambda$$

$$f \cong \frac{3 \cdot 10^{10} \text{ cm/sec}}{10^{-4} \text{ cm}}$$

$$f = 3 \cdot 10^{14}/\text{sec} \ldots \textit{whew}!!!$$

$$E = 6.6 \cdot 10^{-34} \times 3 \cdot 10^{-14} = 1.98 \cdot 10^{-19} \text{ joules}$$

This sounds small, which it is according our everyday scale. However, it is very close to the energy that an electron would have if it were accelerated through a potential of one volt:

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ coul} \times 1 \text{ V} = 1.6 \cdot 10^{-19} \text{ joule}$$

In photonics, the typical energies that you work with involve electrons in a potential of 1 or 2 V. So we use the energy of an electron accelerated through a potential of 1 V as a handy unit—the electron volt (eV).

The energy of a photon with a wavelength of 1000 nm (or 1 μm) is

$$E = \frac{1.98 \cdot 10^{-19}}{1.6 \cdot 10^{-19}} = 1.24 \text{ eV} \tag{2.8}$$

It is easy to show that reverse is true. That is, a photon with an energy of 1 eV has a wavelength of 1.24 μm (= 1240 nm). If a photon with a wavelength of 1 μm has an energy of 1.24 eV, what is the energy of a photon having a wavelength of 0.5 μm (= 500 nm)? Answer: E = 2.48 eV.

What is the energy of red photons (λ = 612 nm)? Answer: E = 2.0 eV.

Exercise 2.3

Prove that the energy of any photon is given by

$$E = \frac{1.24 \text{ } \mu\text{m}}{\lambda} \text{ eV} \tag{2.9}$$

Prove that the wavelength of any photon is given by

$$\lambda = \frac{1.24 \text{ eV}}{E} \text{ } \mu\text{m} \tag{2.20}$$

Since photons always travel at the speed of light, it is natural to think about the flow of energy or power in a light beam. Power is measured in watts:

Watts = power that comes out of the light bulb = energy/sec

Watts = number of photons of frequency f/sec
× energy, summed over all f

$$\text{Power} = \sum_f n_f \cdot E_f$$

So the total power is made up of the sum of all these little packets of

$$E = hf$$

It is sometimes more convenient in many applications to use angular frequency ω instead of regular frequency:

$$\omega = 2\pi f$$

To make everything work out right you have to divide Planck's constant by 2π:

$$h/2\pi \rightarrow \hbar$$

$$E = \hbar\omega$$

In photonics, you will use λ and E almost always. Rarely will you calculate f. The most important reason for this is experimental in origin. There are no instruments that measure frequency of photons directly.

2.4   Properties of Electrons

Electrons are the *ONICS* of phot*ONICS*. Electrons can interact with photons one at a time (mostly) through the medium of a semiconductor crystal. When a semiconductor absorbs a photon, the energy of the photon can be transferred to an electron as potential energy. When the electron loses potential energy, the semiconductor can account for the energy difference by emitting a photon.

Exercise 2.4

A photon with energy 1.5 eV strikes GaAs. The energy is absorbed by breaking one bond, promoting one electron from a bonding state (valence band) to an antibonding state (conduction band), and leaving a vacant state (hole) in the valence band. Some time later, the electron recombines with the hole, completing the bond and releasing a photon of 1.42 eV, the bonding energy of GaAs at room temperature.

An electron can be characterized by its mass, charge and magnetic moment, all of which are fixed in magnitude. It is also characterized by its energy and momentum, which are variable. Although the electron does not have a well-defined size, it behaves in many respects as a particle. For example, we could write down expressions for the momentum and energy of a baseball:

$$\text{momentum} = mv = p$$

$$\text{kinetic energy} = \frac{1}{2}\, mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \tag{2.21}$$

The same thing is true for electrons. Photons, of course, don't have any mass. So this equation does not work for photons.

A graph of the energy of a free electron as a function of its momentum, just like that of a baseball, is a parabola (see Fig. 2.3). Remember that a 1 eV photon has $\lambda = 1240$ nm.
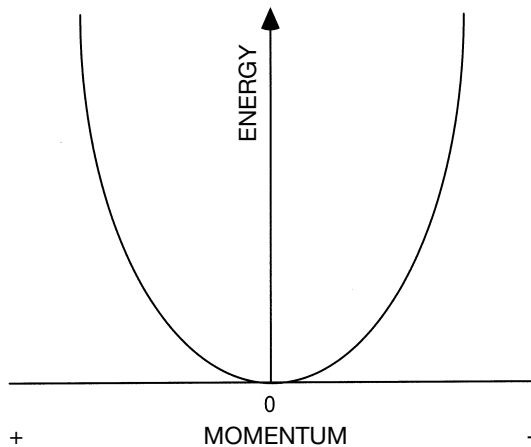
On the other hand, we know from Maxwell's equations that photons do have a momentum that is equal to

$$p = \frac{E}{c} = \frac{hf}{c} \tag{2.22}$$

But, since $c = f\lambda$,

$$p = \frac{h}{\lambda} = \hbar k, \qquad \text{where} \qquad k = \frac{2\pi}{\lambda} \tag{2.23}$$

So, photons don't have mass, but they have momentum.

Figure 2.3. The kinetic energy of a particle with mass, like that of an electron, is proportional to the square of its momentum.

Electrons have momentum, but can they have a wavelength? Well if your name were Prince Louis-Victor, Duke de Broglie, and the year was 1924, maybe such an idea would not seem so strange. If this were the case, then the energy of an electron would be

$$E = \frac{1}{2m} \cdot \frac{h^2}{\lambda^2}$$

Using this equation, you could actually calculate the wavelength if you knew the electron energy. Suppose your electron has an energy of 1 eV. This is the energy of an electron that falls through a potential of 1 V.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules}$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6 \times 10^{-34} \text{ joule-sec}}{\sqrt{2 \cdot 9 \times 10^{-31} \text{ kg} \cdot 1.6 \times 10^{-19} \text{ joules}}} = 12 \text{ Å}$$

In 1929, de Broglie received the Nobel prize for this revolutionary idea. His reasoning was different from the simple analysis above, and involved little math, not to mention Maxwell's equations. His insight was based on an analogy with his everyday experience and is presented later on in Section 2.6. Nearly ten years later, in 1937, the Nobel prize was awarded to Clint Davisson for his observation of electron diffraction, a property of electrons that can be described only by its fundamental wave-like nature. His lab partner, Lester Germer, got left out of the prize list, a mystery to this day.

The work of Davisson and Germer led directly to the invention of the electron microscope, a widely used instrument in all branches of materials physics and engineering.

$$\text{For a 1 eV photon, } \lambda = 12{,}400 \text{ Å}$$

$$\text{For a 1 eV electron, } \lambda = 12 \text{ Å}$$

$$\text{At 1 eV energy (only), } \frac{\lambda_{\text{photon}}}{\lambda_{\text{electron}}} = 1000$$

This ratio depends on the electron energy. But 1 eV is characteristic of electrons in solids. What does this mean?

Relative to the electron, the photon has mostly energy, but not very much momentum. We can see this on the diagram of energy and momentum (Fig. 2.4).

Except for the uninteresting case in which $E = 0$, the energy momentum curves for free electrons and photons do not intersect. That is: there is no point on the curves where the energy and momentum of an electron are equal to the energy and momentum of a photon. This
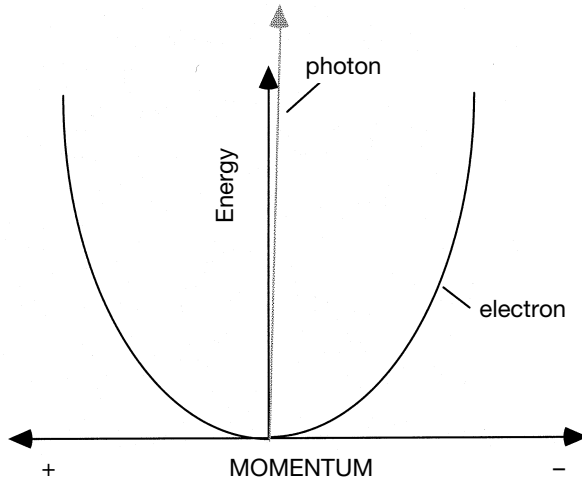
Figure 2.4. The energy of a photon is linearly proportional to its momentum. When plotted on the same graph as that for an electron, the energy–momentum relationship for a photon looks like a vertical line.

means that a free electron and a photon cannot interact with each other. However, in a solid material the situation is different. Electrons and photons can interact because the host material can supply the momentum that is missing in the case of a free electron and a photon. This is discussed in more detail in Section 2.7.

Imagine a vapor of single atoms of the same element. Before atomic bonding occurs, the constituent atoms are "free" to wander around. They are in an antibonding state. We could take silicon as an example. When two such free silicon atoms meet, they may bond together. They will do so because the bonding state is at a lower energy than what existed previously. The valence electrons have thus fallen into some kind of potential well, and to do so they gave up some of their energy. This energy that separates the bonding state from the higher energy antibonding state is called the *bonding energy*. In silicon, this energy difference is about 1 eV.

If a photon comes along, or if the thermal energy is large enough, one of those bonds might happen to break and now there would be an electron that is promoted from the bonding state to the antibonding state. Of course, if all the bonds were broken the silicon would melt. But what does the situation look like for us? At room temperature in perfect silicon are there any broken bonds? How could you estimate this?