Refraction

Geometrical optics uses rays to represent light propagation through an optical system. According to Fermat’s principle, rays will choose a path that is stationary with respect to variations in path length, typically a time minimum. The quickest path may result in a change of direction at the interface between two refractive media separated by a refracting surface.

The change of direction is described by Snell’s law,

\[ n' \sin I' = n \sin I \]

\( n \) and \( n' \) are the refractive indices of the first and second refractive media. The refractive index describes the speed of light in vacuum relative to the medium. \( I \) is the angle of incidence, and \( I' \) is the angle of refraction. Both are measured relative to the surface normal. For rotationally symmetric systems, the optical axis (OA) is the axis of symmetry taken along the \( z \) direction.

Image formation requires a spherical refracting surface so that an incident ray travelling parallel to the OA can be bent towards the OA. However, spherical surfaces give rise to aberrations and associated image defects. A conceptually simple example is spherical aberration, where rays originating from a unique object point do not converge to form a unique image point at the OA.
Gaussian Optics

Gaussian optics treats each surface as free of aberrations by performing a linear extension of the paraxial region to arbitrary heights above the OA.

Since tan $\theta = \theta$ holds exactly in the paraxial region, each refracting surface is projected onto a flat tangent plane normal to the OA, and the surface power $\Phi_s$ is retained. The angles $i$ and $i'$ are now equivalent to $u$ and $u'$, where

$$u = \tan^{-1}\left(\frac{y}{s}\right) = \frac{y}{s}, \quad u' = \tan^{-1}\left(-\frac{y}{s'}\right) = -\frac{y}{s'}$$

$u$ and $u'$ are interpreted as ray slopes instead of angles.

Substituting $u$ and $u'$ into the Gaussian conjugate equation for a spherical surface yields a new form of Snell’s law,

$$n'u' = nu - y\Phi_s$$

Paraxial imaging is now valid at arbitrary heights $h, h'$.

OP is the object plane, and IP is the image plane.
Aberrations

When rays are traced using exact trigonometric equations based on Snell’s law (p. 1), aberrations arise from the higher-order terms in the expansion of the sine function:

\[ \sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots \]

Primary (Seidel) aberrations arise from the third-order term:

- **Spherical aberration** (SA) is caused by variations of focus with ray height in the aperture (p. 26). A converging element has under-corrected SA, so rays come to a focus closer to the lens as the ray height increases.
- **Coma** is caused by variation of magnification with ray height in the aperture so that rays from a given object point will be brought to a focus at different heights on the IP. The image of a point is spread into a non-symmetric shape that resembles a comet. Coma is absent at the OA but increases with radial distance outwards.
- **Astigmatism** arises because rays from an off-axis object point are presented with a tilted rather than symmetric aperture. A point is imaged as two small perpendicular lines. Astigmatism improves as the aperture is reduced.
- **Petzval field curvature** describes the fact that the IP corresponding to a flat OP is itself not naturally flat. Positive and negative elements introduce inward and outward curvature, respectively. Field curvature increases with radial distance outwards.
- **Distortion** describes the variation of magnification with radial height on the IP. Positive or pincushion distortion will pull out the corners of a rectangular image crop to a greater extent than the sides, whereas barrel distortion will push them inwards. Distortion does not introduce blur and is unaffected by aperture.
- **Chromatic aberration** and lateral color appear in polychromatic light due to variation of focus with wavelength \( \lambda \) and off-axis variation of \( m \) with \( \lambda \), respectively.

Aberrations affect image quality and can be measured as deviations from the ideal Gaussian imaging properties. Modern compound lenses are well corrected for primary aberrations.
Autofocus

Phase-detect autofocus (PDAF) systems used in digital SLR cameras have evolved from the 1985 Minolta Maxxum design. The reflex mirror has a zone of partial transmission, and the transmitted light is directed by a secondary mirror down to an autofocus (AF) module located at an optically equivalent SP (p. 14). The module contains microlenses that direct the light onto a CCD strip.

Consider light passing through the lens from a small region on the OP indicated by an AF point on the focusing screen. When the OP is in focus at the SP, the light distribution arriving from equivalent portions of two halves of the XP will produce an identical optical image and signal along each half of the CCD strip.

When the OP is not in sharp focus (i.e., out of focus), these images will shift either toward or away from each other, and this shift indicates the direction and amount by which the lens needs to be adjusted by its AF motor.

A horizontal CCD strip is suited for analyzing signals with scene detail present in the horizontal direction, such as the change of contrast provided by a vertical edge. A cross-type AF point utilizes both a horizontal and a vertical CCD strip so that scene detail in both directions can be utilized to achieve focus.
When focus is set at infinity, \( s' = f' \). The ray slope is then
\[
u' = \frac{D}{2f'}
\]
\( u' \) can be substituted into the formula for the RA (p. 29). The refractive indices can be removed using \( n'/n = f'/f \).

The illuminance \( E \) at the axial position on the SP becomes
\[
E = \frac{\pi}{4} LT \frac{1}{N^2}, \quad \text{where} \quad N = \frac{f}{D}
\]

The f-number \( N \) depends on the front effective focal length and the EP diameter \( D \).

The f-number is the reciprocal of the RA when focus is set at infinity. It is usually marked on lens barrels using the symbols \( f/N \) or \( 1:N \), where \( N \) is the f-number.

Beyond Gaussian optics, the f-number can be written
\[
N = \frac{n}{2NA'_{\infty}}, \quad \text{where} \quad NA'_{\infty} = n' \sin U'
\]

\( NA'_{\infty} \) is the image-space numerical aperture when \( s \to \infty \), and \( U' \) is the real image-space marginal ray angle. Provided the lens is aplanatic (free from SA and coma), then \( \sin U' = u' \) when \( s \to \infty \), according to Abbe’s sine condition, so the Gaussian expression \( N = f/D \) is exact for an aplanatic lens. However, the sine function restricts the lowest achievable value in air (\( n = n' = 1 \)) to \( N' = 0.5 \).
Color Filter Arrays

The HVS is sensitive to wavelengths between 380–780 nm. The eye contains three types of cone cells with photon absorption properties described by a set of eye cone response functions \( \tilde{l}(\lambda), \tilde{m}(\lambda), \) and \( \tilde{s}(\lambda) \). These different responses lead to the visual sensation of color (p. 53).

A larger number of green filters are used since the HVS is more sensitive to \( \lambda \) in the green region of the visible spectrum.

A camera requires an analogous set of response functions to detect color. In consumer cameras, a color filter array (CFA) is fitted above the imaging sensor.

The Bayer CFA uses a \( 2 \times 2 \) block pattern of red, green, and blue filters that form three types of mosaic. The filters have different spectral transmission properties described by a set of spectral transmission functions \( T_{CFA,i}(\lambda) \), where \( i \) is the mosaic label. The overall camera response is determined largely by the product of \( T_{CFA,i}(\lambda) \) and the charge collection efficiency \( \eta(\lambda) \) (pp. 37, 40).

The Fuji® X-Trans® CFA uses a \( 6 \times 6 \) block pattern. It requires greater processing power and is more expensive but can give improved image quality.

- An infrared-blocking filter is combined with the CFA to limit the response outside of the visible spectrum.
- The spectral passband of the camera describes the range of wavelengths over which it responds.
- In order to record color correctly, a linear transformation should in principle exist between the camera response and eye cone response functions.
- After raw data capture, only the digital value of one color component is known at each photosite. Color demosaicing estimates the missing values so that all color components are known at every photosite.
Color Theory

Color vision is a perceived physiological sensation to electromagnetic waves with wavelengths in the visible region, which ranges from approximately 380–780 nm.

Color can be described by its luminance and chromaticity. Chromaticity can be subdivided into hue and saturation components. Pure spectrum colors (rainbow colors) are fully saturated. Their hues can be divided into six main regions. Each region contains many hues, and the transitions between regions are smooth.

Colors that are not pure spectrum colors have been diluted with white light and are not fully saturated. Pink is obtained by mixing a red hue with white. Greyscale or achromatic colors are fully desaturated.

Light is generally polychromatic, i.e., a mixture of various wavelengths described by its spectral power distribution (SPD) $P(\lambda)$. This can be any function of power at each $\lambda$, for example, spectral radiance (p. 39). Metamers are different SPDs that appear to be the same color under the same viewing conditions. The principle of metamerism is fundamental to color theory.
In 1931, before the eye cone response functions were known, the CIE (Commission internationale de l’éclairage) analyzed results from color-matching experiments that used a set of three real monochromatic primaries:

\[ \lambda_R = 700 \text{ nm}, \quad \lambda_G = 546.1 \text{ nm}, \quad \lambda_B = 435.8 \text{ nm} \]

Using Grassmann’s laws, human observers were asked to visually match the color of a monochromatic light source at each wavelength \( \lambda \) by mixing amounts of the primaries. From the results, the CIE defined a set of color-matching functions (CMFs) denoted \( \bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda) \).

Since the primaries are necessarily real, negative values occur. This physically corresponds to matches that required a primary to be mixed with the source light instead of the other primaries. The CMFs are normalized such that the area under each curve is the same; adding a unit of each primary matches the color of a unit of a reference white that was chosen to be CIE illuminant E. This hypothetical illuminant has constant power at all wavelengths.

Adding equal photometric or radiometric amounts of the primaries does not yield the reference white; the actual luminance and radiance ratios between a unit of each primary are 1 : 4.5907 : 0.0601 and 72.0966 : 1.3791 : 1.

CIE RGB tristimulus values can be defined, and each \( R, G, B \) triple defines a color in the CIE RGB color space. This is again a reference color space. The CMFs and \( \bar{l}(\lambda), \bar{m}(\lambda), \bar{s}(\lambda) \) are linearly related (p. 53). A different set of primaries would give a different set of CMFs obtainable via a linear transformation.
White Balance: Matrix Algebra

The raw to sRGB transformation (p. 61) for scene illumination with a D65 white point can be rewritten:

\[
\begin{bmatrix}
R_L \\
G_L \\
B_L
\end{bmatrix}_{D65} = M_R D_{D65} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{D65}
\]

where \( M_R D_{D65} = M_{-1}^{sRGB} C \)

\( M_R \) is a color rotation matrix (each row sums to 1):

\[
M_R = M_{-1}^{sRGB} C D_{D65}^{1}
\]

\( D_{D65} \) is a diagonal white balance matrix of raw WB multipliers for D65 scene illumination:

\[
D_{D65} = \begin{bmatrix}
1/R_{WP} & 0 & 0 \\
0 & 1/G_{WP} & 0 \\
0 & 0 & 1/B_{WP}
\end{bmatrix}_{D65}
\]

\( 1/G_{WP} = 1 \) provided \( C \) has been appropriately normalized (p. 59). Typically, \( 1/R_{WP} > 1 \) and \( 1/B_{WP} > 1 \).

For scene illumination with a different white point, \( D_{D65} \) can be replaced by a matrix suitable for the adopted white:

\[
\begin{bmatrix}
R_L \\
G_L \\
B_L
\end{bmatrix}_{D65} = M_R D_{AW} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}_{AW} ; D_{AW} = \begin{bmatrix}
1/R_{WP} & 0 & 0 \\
0 & 1/G_{WP} & 0 \\
0 & 0 & 1/B_{WP}
\end{bmatrix}_{AW}
\]

Better accuracy can be achieved using \( M_R \) derived from a characterization performed with illuminant CCT closely matching that of the AW. Camera manufacturers typically use a set of rotation matrices, each optimized for use with an associated WB preset. Two presets from the Olympus® E-M1 raw metadata are tabulated below (divide by 256).

<table>
<thead>
<tr>
<th>CCT</th>
<th>Scene</th>
<th>Multipliers</th>
<th>( M_R )</th>
</tr>
</thead>
</table>
| 3000 K | Tungsten | 296, 256, 760 | \[
\begin{bmatrix}
324 & -40 & -28 \\
-68 & 308 & 16 \\
16 & -248 & 488
\end{bmatrix}
\] |
| 6000 K | Cloudy  | 544, 256, 396 | \[
\begin{bmatrix}
380 & -104 & -20 \\
-40 & 348 & -52 \\
10 & -128 & 374
\end{bmatrix}
\] |
Image Display Resolution

The pixel count of a digital image is the total number of pixels. It is often expressed as $n = n_h \times n_v$, where $n_h$ and $n_v$ are the horizontal and vertical pixel counts. The image display resolution is the number of displayed image pixels per unit distance, e.g., pixels per inch (ppi). It is a property of the display device/medium:

1) For a monitor/screen, the image display resolution is defined by the screen resolution, which typically has a value such as 72 ppi, 96 ppi, etc.

2) For a hard-copy print, the image display resolution can be chosen by the user; 300 ppi is considered enough for high-quality prints viewed at $D_v$.

Note that image display resolution is independent of the printer resolution, which is the number of ink dots per unit distance used by a printer, e.g. dots per inch (dpi). For a given printer technology, a larger dpi can yield a better-quality print. The image display size is defined as

$$\text{image display size} = \frac{\text{pixel count}}{\text{image display resolution}}$$

The print size is the image display size for a print.

Example: The image display size for a digital image viewed on a 72-ppi monitor will be $300/72 \approx 4.16$ times larger than that of a 300-ppi print of the same image.

Images cannot be “saved” at a specific ppi since image display resolution is a property of the display, but a tag can be added to the image metadata indicating a desired ppi for the printer software. This tag does not affect the image pixel count, and the value can be overridden.

In order to match a desired image display size for a given image display resolution (or vice versa), the pixel count needs to be changed through image resampling. Printer software will automatically perform the required resampling when printing an image. Notably, this does not change the pixel count of the stored image file. Image editing software can be used to resample an image. In contrast to printing, the resampled image needs to be resaved, which will alter its pixel count.
Exposure Value

The reflected and incident-light metering equations (pp. 75, 82) can be rewritten as the APEX equation (additive system of photographic exposure) designed to simplify manual calculations:

\[ Ev = Av + Tv = Bv + Sv = Iv + Sv \]

- \( Ev \) is the exposure value (not to be confused with \( E_v \)).
- \( Av = \log_2 N^2 \) is the aperture value.
- \( Tv = -\log_2 t \) is the time value.
- \( Bv = \log_2 (\langle L \rangle/(0.3K)) \) is the brightness value.
- \( Iv = \log_2 (E/(0.3C)) \) is the incident-light value.
- \( Sv = \log_2 (S/3.125) \) is the speed value.

These are associated with specific \( N, t, \langle L \rangle, S \). For example,

<table>
<thead>
<tr>
<th>( N )</th>
<th>0.5</th>
<th>0.7</th>
<th>1</th>
<th>1.4</th>
<th>2</th>
<th>2.8</th>
<th>4</th>
<th>5.6</th>
<th>etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Av )</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t )</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>1/2</th>
<th>1/4</th>
<th>1/8</th>
<th>1/16</th>
<th>1/32</th>
<th>etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Tv )</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

\( Bv \) and \( Iv \) depend upon the choice of reflected light and incident light meter calibration constants \( K \) and \( C \). Suitable exposure settings for a typical scene are provided by any combination of \( N, t, S \) that yields the recommended \( Ev \). A difference of 1 \( Ev \) defines a photographic stop (pp. 34, 45). An f-stop specifically relates to a change of \( Av \).

Exposure compensation (EC, p. 81) can be included in the APEX equation by modifying the brightness value:

\[ Bv \rightarrow Bv - EC \]

Positive EC compensates when \( Bv \) is too high by reducing \( Ev \) and therefore increasing \( \langle H \rangle \). Negative EC compensates when \( Bv \) is too low by raising \( Ev \) and therefore decreasing \( \langle H \rangle \).

The APEX equation is valid provided the metering is based on average photometry. It is not valid when using in-camera matrix/pattern metering modes (p. 82).
High Dynamic Range: Example

Due to back lighting, negative EC was required to preserve the highlights in the sun. This rendered the shadows much too dark. The shutter speed was $t = 1/2500$ s.

A frame taken at +2 Ev using a shutter speed $t = 1/640$ s reveals more shadow detail but clips the highlights.

A frame taken at +4 Ev using a shutter speed $t = 1/160$ s completely clips the highlights in order to reveal the full shadow detail.

Merging the three raw files into a linear HDR image and then applying a local TMO yields an output image with both highlight and shadow detail visible.
Polarizing Filters: Practice

The utility of the polarizing filter is that the ratio between unpolarized light and any partially or completely plane polarized light entering the lens can be altered. Dielectric surfaces from which partially or completely plane polarized light emerges include glass, wood, leaves, paint, and water. Rotating the filter plane of transmission to eliminate plane polarized light emerging from glass or water can eliminate the observed reflections. Eliminating reflections from leaves can reveal their true color.

Light arriving from clouds will in general be unpolarized due to repeated diffuse reflections (p. 99) and will be unaffected by a polarizing filter. Light arriving from a blue sky will in general be partially plane polarized due to Rayleigh scattering from air particles (pp. 99, 101). Therefore, rotating the filter to reduce the partially plane polarized light will darken the blue sky relative to any sources of unpolarized light. The darkening effect is most noticeable along the vertical strip of sky that forms a right angle with the sun and the observer since maximum polarization occurs at a 90° scattering angle from the sun.

- A plane or linear polarizing filter should not be used with digital SLR cameras as it will interfere with the autofocus and metering systems. These utilize a beamsplitter that functions using polarization.
- A circular polarizing filter (CPL) can be used with digital SLR cameras in place of a linear polarizing filter. A CPL functions as an ordinary linear polarizer but modifies the transmitted (plane polarized) light so that $\mathbf{E}$ rotates as a function of time and traces out a helical path. This prevents the transmitted light from entering the beamsplitter in a plane polarized state.

A polarizing filter should be removed from the lens in low light conditions since an ideal polarizing filter only transmits 50% of all unpolarized light, equivalent to use of a 1-stop neutral density filter (p. 88).
Side Lighting: Example

Luoping, Yunnan, China
Side lighting reveals the depth and 3D form.

Llyn Gwynant, Snowdonia, Wales
Side lighting reveals the surface texture.
Sync Speed

The shutter speed $t$ is typically set much slower than the flash duration itself when using flash. Shutter speeds quicker than the sync speed should not be used, otherwise image artifacts can arise due to conflict between the flash and the shutter method of operation.

**Mechanical focal plane shutter**: Since very quick $t$ are obtained only when the second shutter curtain starts to close before the first curtain fully opens (p. 35), the sync speed is the quickest $t$ at which the first curtain can fully open before the second curtain needs to start closing, on the order of $1/250$ s. This ensures that the flash can be fired when both curtains are fully open. Quicker $t$ would shield part of the scene from the flash and cause a dark band to appear in the image.

**Mechanical leaf shutter**: Since this is positioned next to the lens aperture, the sync speed is limited only by the quickest possible $t$, typically on the order of $1/2000$ s.

**Electronic global shutter and CCD sensor**: The sync speed is limited only by the quickest electronic shutter speed available or by the flash duration.

**Electronic rolling shutter and CMOS sensor**: Although quicker shutter speeds $t$ are available compared to a mechanical shutter, exposing all sensor rows to the flash requires limiting the sync speed to the total frame readout time due to the rolling nature of the shutter (p. 35). This is typically too slow to be useful.

**Electronic global shutter and CMOS sensor**: Available on scientific cameras, the sync speed is limited only by the quickest electronic $t$ or by the flash duration.

If a $t$ faster than the sync speed is required with a focal plane shutter, e.g., when a low $N$ is needed for shallow DoF, the high-speed sync (HSS) mode fires the flash continuously at low power throughout the entire duration $t$ in order to eliminate shielding artifacts. In this case, the effective flash duration and $t$ are the same. Nevertheless, “high speed” refers to shutter speed $t$ and not the effective flash duration; high-speed photography requires a single flash of very short duration, such as $1/32000$ s.
Camera System MTF

The **camera system MTF** is the product of all the individual component MTFs (p. 110):

\[
|H(\mu_x, \mu_y)| = |H_1(\mu_x, \mu_y)| |H_2(\mu_x, \mu_y)| \cdots |H_n(\mu_x, \mu_y)|
\]

Although a huge variety of component MTFs contribute to the camera system MTF, a basic system can be defined using the diffraction, detector aperture, and optical low-pass filter (OLPF) components.

The camera system MTF depends greatly upon the diffraction MTF since this varies with lens f-number \(N\).

**Example** (see p. 115): \(d_x = 3.8 \, \mu \text{m}, p_x = 4.0 \, \mu \text{m};\) \(\mu_{c,\text{det}} = 263 \, \text{cycles/mm}, \mu_{x,\text{Nyq}} = 125 \, \text{cycles/mm}.\)

- \(N = 22:\) Here the diffraction MTF dominates. The OLPF is not required at this f-number (also see p. 118).

- \(N = 1.4:\) Here, the OLPF MTF dominates. The detector aperture MTF would dominate without the OLPF, in which case aliasing would occur (see pp. 117, 118).