Probability Density Function

If $x$ is a \textbf{continuous RV}, its \textbf{probability density function (PDF)} is related to its CDF by

$$f_x(x) = \frac{dF_x(x)}{dx}$$

Thus, the CDF can also be recovered from the PDF via integration, i.e.,

$$F_x(x) = \int_{-\infty}^{x} f_x(u)du$$

The shaded area in the figure represents the CDF; hence,

$$\Pr(a < x \leq b) = F_x(b) - F_x(a) = \int_{a}^{b} f_x(u)du$$

Because the probability $F_x(x)$ is nondecreasing, it follows that

$$f_x(x) \geq 0, \quad -\infty < x < \infty$$

Also, by virtue of axiom 2, we see that

$$\int_{-\infty}^{\infty} f_x(x)dx = 1$$

That is, the total area under the PDF curve is always unity.

For a \textbf{discrete RV} $x$ that takes on values $x_k$ with probabilities $\Pr(x = x_k), k = 1, 2, 3, \ldots$, it follows that

$$F_x(x) = \sum_{k=1}^{\infty} \Pr(x = x_k)U(x - x_k), \quad f_x(x) = \sum_{k=1}^{\infty} \Pr(x = x_k) \delta(x - x_k)$$

where $U(x - a)$ is the unit step function, and $\delta(x - a) = dU(x - a)/dx$ is the Dirac delta function.