

CHAPTER 6

Prism Design

Many types of prisms have been designed for use in various optical instrument applications. Most have unique shapes as demanded by the geometry of the ray paths, reflection and refraction requirements, and compatibility with manufacture, weight reduction considerations, and provisions for mounting. Before we consider how to mount these prisms, we should understand how they are designed. Our first topics in this chapter are the functions of prisms, geometric relationships that govern those functions, refractive effects, total internal reflection, and the construction and use of tunnel diagrams. We then see how to determine aperture requirements and reference analytical means for calculating third-order aberration contributions from prisms. The chapter closes with design information for 30 types of individual prisms and prism combinations frequently encountered in optical instrument design.

6.1 Principal Functions

The principal functions or uses of prisms (and of some mirrors) are as follows:¹

- To bend (deviate) light around corners,
- To fold an optical system into a given shape or package size,
- To provide proper image orientation,
- To displace the optical axis,
- To adjust optical path length,
- To divide or combine beams by intensity or aperture sharing at a pupil,
- To divide or combine images at an image plane,
- To dynamically scan a beam,
- To disperse light spectrally, and
- To modify the aberration balance of the system of which they are a part.

Some prisms accomplish more than one function simultaneously.

6.2 Geometric Considerations

6.2.1 Refraction and reflection

The laws of refraction and reflection of light govern the passage of rays through prisms and mirrors. In Fig. 6.1, we see a comparison of ray paths from an object passing through a lens and a reflector (mirror) en route to the image. In Fig. 6.1(a), the reflector is a flat mirror while in Fig. 6.1(b); it is a right-angle prism where reflection occurs at an internal surface. The most significant differences are the ray deviations that occur at the prism's refracting surfaces and the axial displacement of the image caused by the replacement of air by glass in part of the path. Refraction in the prism, of course, follows Snell's law, which may be written as:

$$n_j \sin I_j = n'_j \sin I'_j, \quad (6.1)$$

where n_j and n'_j are the refractive indices in object and image spaces of surface “ j ” and I_j and I'_j are the ray angles of incidence and refraction, respectively.

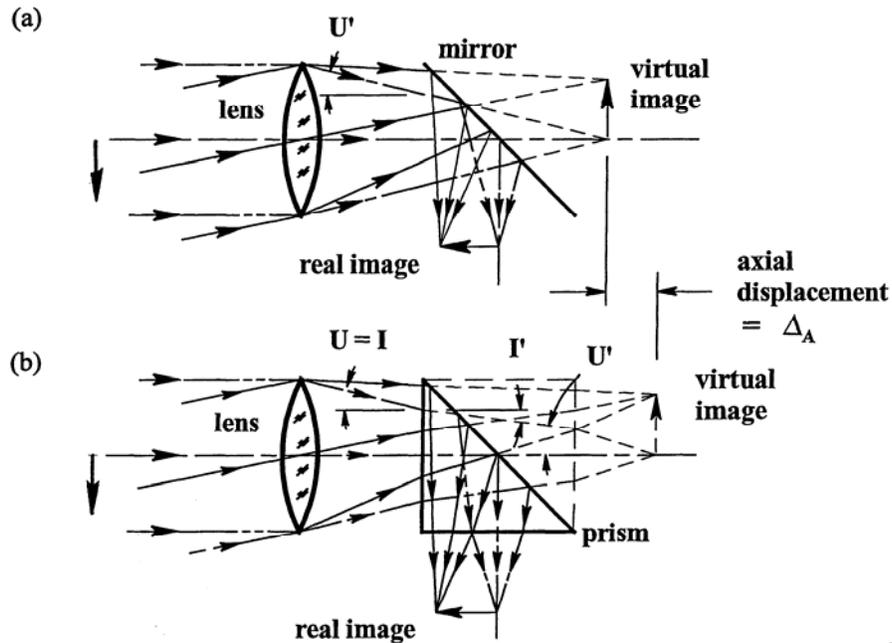


Figure 6.1 90-deg deviations by reflection of rays, (a) at a 45-deg mirror and (b) in a right-angle prism. In (b), angles U , U' , I , and I' pertain to the first surface of the prism.

Reflection follows the familiar relationship

$$I'_j = I_j, \quad (6.2)$$

where I_j and I'_j are the angle's incidence and reflection at surface “ j ”. The angles in these equations are measured with respect to the surface normal at the point of incidence of the ray on the surface. A change in algebraic sign of the ray angle occurs upon reflection. The algebraic signs of the angles are not shown in either of these equations.

The entrance and exit faces of most prisms are oriented perpendicular to the optical axis of the optical system. This promotes symmetry and reduces aberrations for noncollimated beams passing through the prism. Notable exceptions are the Dove prism, the double-Dove prism, wedge prisms, and prisms used to disperse light in monochromators and spectrographs.

A prism with faces normal to the optical axis refracts rays exactly as would a plane parallel plate oriented normal to the axis. The geometrical path length, t_A , through the prism measured along the axis is the same as the thickness of the plate. Any reflections occurring inside the prism do not affect this behavior. The axial displacement, Δ_A (see Fig. 6.1), of an image formed by rays passing through the prism is given by

$$\Delta_A = t_A \left[\frac{(\tan U')}{\tan U} \right] = \left(\frac{t_A}{n} \right) \left[n - \left(\frac{\cos U}{\cos U'} \right) \right]. \quad (6.3)$$

For small angles, this equation reduces to the paraxial version:

$$\Delta_A = \frac{(n-1)t_A}{n}. \quad (6.4)$$

Example 6.1: Image axial displacement due to insertion of a prism. (For design and analysis, use File 6.1 of the CD-ROM)

Assume that a converging lens images a distant object with an $f/4$ beam. How much does the image move axially (a) exactly and (b) paraxially, when a right-angle prism made of FN11 glass with thickness $t_A = 38.100$ mm (1.500 in.) is inserted in the beam?

(a) The marginal ray angle for this $f/4$ beam is

$$\sin U' = \frac{0.5}{f / \text{number}} = \frac{0.5}{4} = 0.1250;$$

hence, $U' = 7.1808$ deg.

From Table B2, the refractive index for FN11 glass is 1.621.

Since the entrance face of the prism is normal to the axis, $I = U'$ so, by Eq. (6.1)

$$\sin I' = \frac{\sin 7.1808 \text{ deg}}{1.621} = 0.07711 \text{ and } I' = 4.4226 \text{ deg.}$$

By Eq. (6.3), the image moves by

$$\Delta_A = \left(\frac{38.100}{1.621} \right) \left[1.621 - \left(\frac{\cos 7.1808 \text{ deg}}{\cos 4.4226 \text{ deg}} \right) \right] = 14.711 \text{ mm (0.579 in.)}$$

(b) By Eq. (6.4), the paraxial approximation of this displacement is

$$\Delta_A = \frac{(1.621-1)(38.1)}{1.621} = 14.596 \text{ mm (0.579 in.)}$$

The reflection within a prism folds the light path. In Fig. 6.1(b), the object (an arrow, not shown) is imaged by the lens through the prism as the indicated virtual image. After reflection, the real image is located as shown. If the page were to be folded along the line representing the reflecting surface, the real image and the solid-line rays would coincide exactly with the virtual image and the dashed-line rays. A diagram showing both the original prism (ABC) and the folded counterpart (ABC') is called a "tunnel diagram" (see Fig. 6.2). The rays $a-a'$ and $b-b'$ represent actual reflected paths, while rays $a-a''$ and $b-b''$ appear to pass directly through the folded prism with proper refraction, but without the reflection. Successive folds of the page represent multiple reflections. This type of diagram, which can be drawn for any prism, is particularly helpful when designing an optical instrument using prisms since it simplifies the estimation of required apertures and hence the sizes of those prisms.

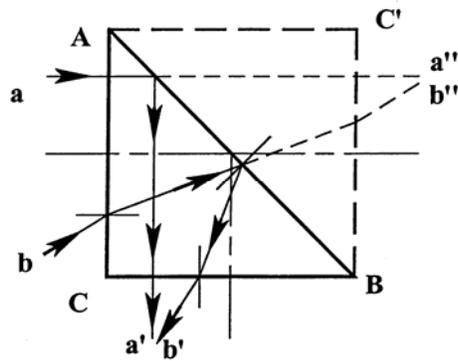


Figure 6.2 Illustration of a tunnel diagram for a right-angle prism.

To illustrate the use of a tunnel diagram, let us consider the telescope optical system of Fig. 6.3. This could be a spotting telescope or one side of a binocular. The Porro prisms serve to erect the image as indicated by the “arrow crossed with a drumstick” symbols at various locations in the figure. Figure 6.4(a) shows the front portion of the same system with the Porro prisms represented by tunnel diagrams. The diagonal lines indicate folds in the light path. We designate all the prism apertures as “ A ”; the axial path length of each prism is then $2A$. In Fig. 6.4(b) the prism path lengths are shown as $2A/n$; these are the thicknesses of air optically equivalent to the physical paths through the prisms. The air-equivalent thickness is sometimes called the “reduced thickness.” In a reduced thickness diagram, the marginal rays converging to the axial image point can be drawn as straight lines (i.e., without refraction). The ray heights at each prism surface (including the

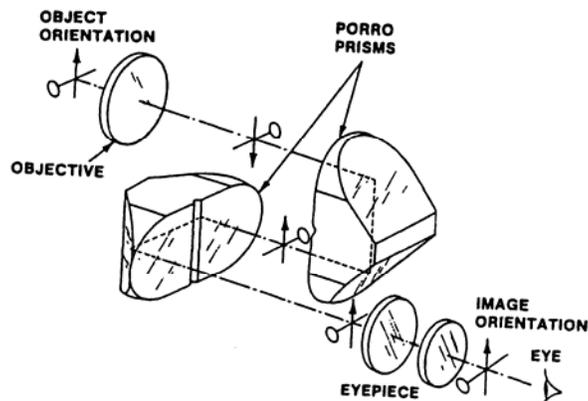


Figure 6.3 Optical system of a typical telescope with a Porro prism erecting system. (From Yoder.¹)

reflecting surfaces) are paraxial approximations of the true values that would be obtained by trigonometric ray tracing. Paraxially, angles in radians replace the sines of the angles. In most applications, this degree of approximation is adequate. For example, an angle of 7 deg is 0.12217 radians and its sine is 0.12187. The differences between these values are not significant for prism design purposes.

Warren Smith used tunnel diagrams to illustrate the determination of the minimum Porro prism apertures required for use in a typical prism erecting telescope.² With a diagram similar to Fig. 6.4(b), he noted that the proportion of face width A_i to reduced thickness was $A_i: (2A_i/n_i)$ or $n_i/2$. He then redrew the diagram in the form shown in Fig. 6.5 to facilitate calculating the minimum value for A_1 and A_2 . The dashed lines drawn from the top front prism corners to the opposite vertices both have slopes, m , equaling one-half the ratio just derived or $n_i/4$. These lines are loci of the corners of a family of prisms with the proper proportions. The intersections of these two dashed lines with the outermost full-field ray (frequently called the “upper rim ray” or URR) locate the corners of the two Porro prisms. Note that the air spaces between optical components must be known for this procedure to succeed.

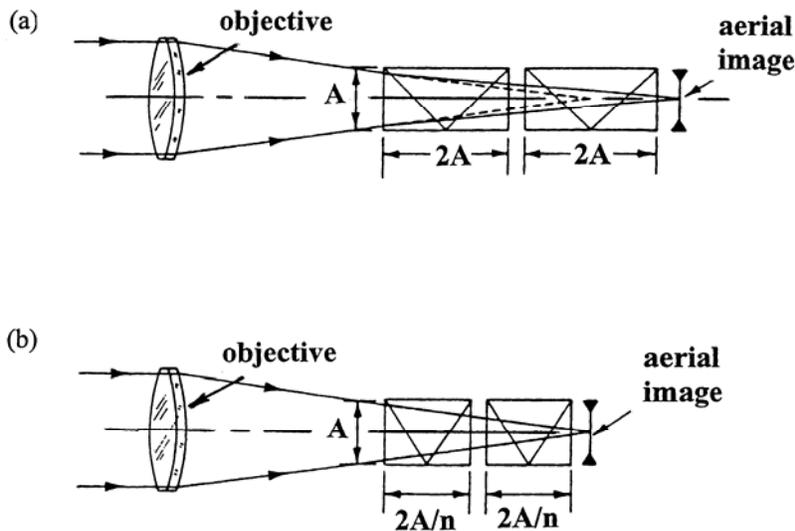


Figure 6.4 Lens and Porro prisms from Fig. 6.3 with prisms shown (a) by conventional tunnel diagrams and (b) by tunnel diagrams with reduced (air equivalent) thicknesses. (Adapted from Smith.²)

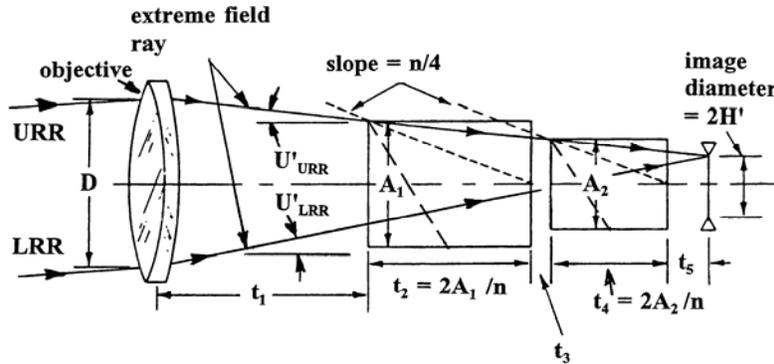


Figure 6.5 Determination of minimum prism apertures from geometric proportions and the outermost unvignetted upper full-field (rim) ray (URR). The corresponding lower field (rim) ray (LRR) is used to determine the ability of the prisms to function by total internal reflection (TIR).

It is easy to see from Fig. 6.5 that the slope of the URR is

$$\tan U'_{URR} = \frac{\left[\left(\frac{D}{2} \right) - H' \right]}{EFL_{OBJ}}, \quad (6.5)$$

and that the semiaperture of the second prism is $A_2/2 = H' + (t_4 + t_5)(\tan U')$. This semiaperture also is given by the expression $A_2/2 = (n_i)(t_4)/4$. Equating these expressions for A_2 , we find that the thickness and aperture of the second prism are

$$t_4 = \frac{[(t_5)(\tan U'_{URR}) + H']}{\left(\frac{n_i}{4} - \tan U'_{URR} \right)}, \quad (6.6)$$

$$A_2 = \frac{(n_i)(t_4)}{2}. \quad (6.7)$$

We can write expressions for the axial thickness and aperture of the first Porro prism as

$$t_2 = \frac{[(t_3 + t_4 + t_5)(\tan U'_{URR}) + H']}{\left(\frac{n_i}{4} - \tan U'_{URR} \right)}, \quad (6.8)$$

$$A_1 = \frac{(n_i)(t_2)}{2}. \quad (6.9)$$

Example 6.2 illustrates the use of this technique.