Multiplying both sides of the **power received equation** by time results in the **energy received equation**, which can then be solved for the **required transmitted energy**:

\[ E_T = E_R \cdot \frac{A_{\text{illuminated}}}{\sigma} \cdot \frac{\pi R^2}{A_{\text{received}}} \cdot \frac{1}{\eta_{\text{atm}}} \cdot \frac{1}{\eta_{\text{sys}}} \]

In this equation it is possible to replace area illuminated with range squared times antenna gain, the gain compared to some reference such as π steradians. The power and energy equations presented so far only calculate how much power (or energy) is received back from a given amount of transmitted power (or energy). This says nothing about how much energy per waveform is needed to detect an object. Thus, for various types of lidars, it is necessary to determine the amount of energy required to achieve a certain combination of probability of detection and probability of false alarm. If a higher probability of false alarm is allowed, a higher probability of detection for a given energy detected can be achieved.

A comparison of the returned signal to multiple noise sources is needed for **direct detection**. If the returned signal is amplified, additional noise is introduced, but the overall **signal-to-noise ratio** (SNR) is usually improved.

The two forms of amplified direct detection lidar are **linear-mode avalanche photodiode** (LMAPD), and **Geiger-mode avalanche photodiode** (GMAPD). GMAPDs are often used with very low probability of detection per pulse, requiring many pulses to achieve the required detection probability.

A GMAPD ladar will often require 50–100 measurements to achieve the required probability of detection and false alarm.

To mitigate noise effects in **heterodyne detection**, instead of amplifying the signal, the power of the LO is increased. However, shot noise depends on the total signal detected, so the LO contributes to additional shot noise.
Four possible GMAPD detection events can trigger the receiver:

1. the detection of a desired target photon,
2. the detection of an undesired foreground clutter photon (such as from backscatter from fog),
3. the detection of undesired background radiation (such as the sun), or
4. the undesired detection of a dark electron.

If many laser pulses are sent out, coincident returns (returns in the same range bin) will result for reflection from a target or from fixed foreground objects. Fog, snow, and rain provide distributed returns over their extent. If the fog or clouds are close enough to the target when an avalanche occurs, the dead time will prevent avalanching on a return from the target.

One problem with GMAPDs is that once the avalanche fires, there is a dead time of at least \(~500\) μsec, during which the GMAPD cannot be armed without a high probability of afterpulsing (unless the arm times are very short, thus reducing the chance of an afterpulse).

The probability of detecting a specified number of photons \(M\) backscattered from the object of interest out of \(N\) pulses can be described by the binomial distribution as follows:

\[
P(M \text{ out of } N \text{ pulses}) = \frac{N!}{M!(N-M)!} (P_o)^M \times (1 - P_o)^{N-M}
\]
If we have multiple high-range-resolution 1D images from different locations, we can develop **angle information** as well as **range information** about the target. The wider the **angular distribution** of the high-range-resolution images the better the angular imaging information. A time-dependent return signal is collected by a single detector to provide a high-range-resolution image. This provides a 1D slice of the 3D spatial profile of the object. When the object rotates or the lidar moves, different slices of the 3D profile can be obtained. If a sufficient number of **1D slices** are collected, a 2D image can be reconstructed.

Various algorithm approaches can be used to combine multiple range profiles to make a more complete image from multiple 1D profile images at different angles. The more complex the object the more necessary it is to obtain range profile information over more and wider angles. Of course, even with a single range profile, 1D imaging can be useful for recognizing some objects, as discussed on page 40.

When a **coherent detection system** is employed and data is collected from multiple views, an object's **Doppler spectrum** can be used to assist in determining the angle from which a given 1D image has been collected. Also, the **rotation speed** of an object can be precisely measured with Doppler techniques. If the object rotates, or the sensor moves, different range projections are formed; thus, knowing the exact rotation speed can make it easier to generate **tomographic images** from multiple 1D images.
For **polarization-based flash lidar imaging** using framing cameras, temporal (range) resolution is provided by a **Pockels cell**, as shown below.

Light enters the receiver. A single polarization of return light is isolated. Alternatively, twice as many cameras can be used to detect both polarizations. A ramp is placed on a Pockels cell to switch polarization as a function of time. Multiple ramps are shown in the figure to depict **unambiguous range**. Two standard framing cameras are used.

In any given detector, the ratio of the power in one camera to that in the other camera provides range information. The primary disadvantage is that a Pockels cell is needed to rotate the polarization, and Pockels cells traditionally require high voltage and have a narrow FOV. A steeper slope provides more accurate **range information** but also has a smaller unambiguous range.

A number of standard techniques can be used to expand the unambiguous range. One of these is **chirping** the length of the ramps. The primary advantage of chirping is that a pair of **standard framing cameras** can be used for **high range resolution**.
Inverse Synthetic Aperture Lidar

**Inverse SAL** (and inverse SAR) is the same as SAL (and SAR), except that the angular diversity is obtained by motion of the object being viewed instead of by motion of the sensor. The same basic physics is at play.

In SAL, the target stays oriented in the same direction, but the lidar moves, creating an angular rotation that allows for angular diversity. The sensor moves a distance $L$ and is a distance $R$ away from the target. The resolution is, again,

$$\Delta \theta = \frac{\lambda}{2L}$$

If the target is a distance $R$, the resolution obtained is $R \times \Delta \theta$. If, instead of the lidar moving, the target rotates,

$$\Delta \phi \approx \frac{L}{R}$$

Here, we have the same angular diversity and can obtain the same resolution. Historically, engineers have tested SARs by looking at targets placed on a rotating platform. The physics is the same as in SALs. The pupil plane field is captured multiple times with the target in multiple orientations. A reference frame adjustment is made, and a larger pupil plane image is created. This image is Fourier transformed to obtain the object plane image of the target.

In SAL, the reference frame adjustment involves adjusting each pupil plane image with the illuminator maintained in the same place. In inverse SAL, the reference frame adjustment involves adjusting each pupil plane image with the illuminator maintained at the same angle with respect to the target.