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A method for measuring two-dimensional distribution of refractive index of a material

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Abstract

The p-polarized light beam is incident on the boundary between a right-angle prism and a test material. When the total internal reflection occurs at the boundary, and the p-polarized light has phase variation. It depends on the refractive index of the tested material. Firstly, the two-dimensional distribution of phase variation of the p-polarized light at the boundary is measured by the four-step phase shifting interferometric technique. Then, substituting the data into the special equations derived from Fresnel equations, the two-dimensional distribution of refractive index of the tested material can be obtained.

Keywords

total internal reflection, phase-shifting interferometry, Fresnel equations.

Summary

A ray of p-polarized light in air is incident at θ_t on the one side surface of a right-angle prism with refractive index n_1, as shown in Fig. 1. The light ray is refracted into the prism and it propagates toward the base surface of the prism. At the base surface of the prism, there is a boundary between the prism and the tested material of refractive index n_2 where $n_1 > n_2$. If θ_i is larger than the critical angle, the light is totally reflected at the boundary. According to Fresnel equations[1], we have

$$n = \sqrt{\frac{-1 + \sqrt{1 + 4(\tan\frac{\phi_p}{2} \cdot \cos\theta_i \cdot \sin\theta_i)^2}}{2(\tan\frac{\phi_p}{2} \cdot \cos\theta_i)^2}}\ , \qquad (1)$$

where $n=n_2/n_1$, and the phase-variation ϕ_p can be written as

$$\phi_p = -2 \cdot \tan^{-1}\left(\frac{\sqrt{\sin^2\theta_i - n^2}}{n^2 \cdot \cos\theta_i}\right). \qquad (2)$$

It is obvious from Eq. (1) that n_2 can be calculated with the measurement of ϕ_p under the experimental conditions in which θ_i and n_1 are specified.

The schematic diagram of this method is shown in Fig. 2. For convenience, the +z-axis is chosen to be along the light propagation direction and the y-axis is along the direction perpendicular to the paper plane. A light coming from a laser light source passes though a polarizer P. If the transmission axis of P is located at 0° with relative to the x-axis, then the light becomes the p-polarized light. A spatial filter S and a lens L collimate the light. The collimating light is incident on a beam splitter BS and divided into two parts: the transmitted light and the reflected light. The reflected light is normally reflected by a mirror M_1driven a piezo-transducer PZT and passes through the BS. Then it enters a CCD camera. Here it acts as the reference light in the interferometer. On the other hand, the transmitted light is reflected by the mirrors M_2 and M_3, and enters a right-angle prism. After it is totally reflected at the boundary between the prism and the tested material, it propagates out of the prism. Then, it is normally reflected by a mirror M_4 and comes back along the original path. It reflected by the BS and enters a

CCD camera. It acts as the test light in the interferometer. The Jones vectors[2] of the reference light and the test light can be written as

$$E_r = a_r \cdot e^{i\phi_r}, \qquad (3a)$$

and

$$E_t = a_t \cdot e^{i\phi_t}, \qquad □3b□$$

respectively, where a_i and ϕ_i (i= t or r) represent the amplitude and the phase. The intensity measured by the CCD is

$$\begin{aligned} I_p &= |E_t + E_r|^2 = a_r^2 + a_t^2 + a_r \cdot a_t \cdot \left(e^{i(\phi_r - \phi_t)} + e^{i(\phi_t - \phi_r)}\right) = a_r^2 + a_t^2 + 2 \cdot a_r \cdot a_t \cdot \cos(2 \cdot \phi_p + \psi) \\ &= A(x,y) + B(x,y)\cos\left(\phi(x,y)\right), \qquad (4) \end{aligned}$$

where A(x,y) and B(x,y) are the intensity coefficients, ϕ_p is the phase variation of the p-polarized light owning to the total internal reflection in the prism, and ψ is the phase difference due to the optical path difference and reflections at BS and mirrors. In order to obtain the distribution of the two-dimensional phase ϕ(x,y), four interferograms[3] are taken by a CCD as the PZT moves M_1 to change the phase of the reference light. The phase π/2 is added between two successive interferograms. So the intensities of these four interferograms can be written as

$$I_i(x,y) = A(x,y) + B(x,y)\cos(\phi(x,y) + \psi_i), \qquad (5)$$

where ψ_i= 0, π/2, π, 3π/2 as i=1, 2, 3, 4, respectively.

By solving these simultaneous equations, we can obtain

$$\phi(x,y) = \tan^{-1}\left(\frac{I_4 - I_2}{I_1 - I_3}\right). \qquad (6)$$

Substituting Eq. (6) into Eq. (4), we have

$$\phi_p(x,y) = \frac{1}{2}(\phi(x,y) - \psi). \qquad (7)$$

In the second measurement let the base surface of the prism free without any test material. We obtain

$$\phi' = 2 \cdot \phi_a + \psi, \qquad (8)$$

where the phase variation ϕ_a can be calculated with the refractive index of a prism n_2 and n_1= 1.

Substituting ϕ_a and ϕ' into the Eq. (8), the data of ψ can be calculated. Then substituting the data of ψ into Eq. (7), ϕ_p(x,y) can be estimated. Finally, the two-dimensional distribution of refractive index of a tested material n_2(x,y) can be evaluated by using Eq. (1b)

In order to show the feasibility of this method, we tested a mixed liquid with ricinus oil, olive oil, baby oil, and water. The refractive indices of three oils and water are 1.513, 1.474, 1.463, and 1.33, respectively. A He-Ne laser with a 632.8 *nm* wavelength and a right-angle prism made of SF8 glass with refractive index n_2= 1.689, were used in this test. The incident angle θ_i was chosen as 69.74°. A 8-bits CCD camera (TM-545, *PULNiX Inc.*) with 510×492 pixels, a PZT (P-830.10, *PI Inc.*), a phase shifter card (DT331, *DT Inc.*), a frame grabber card (Meteor-II/Standard, *Matrox Inc.*), and the analysis software *IntelliWave*^{TM} (*Engineering Synthesis Design Inc.*) were used to drive M_1 and to process interferogram analysis[4,5]. The two-dimensional phase variation distribution and the associated two-dimensional refractive index distribution of the tested material are shown in Fig. 3 and Fig. 4.This method has some merits such as simple optical setup, easy operation and repid measurement. Its validity has been demonstrated.

Acknowledgments

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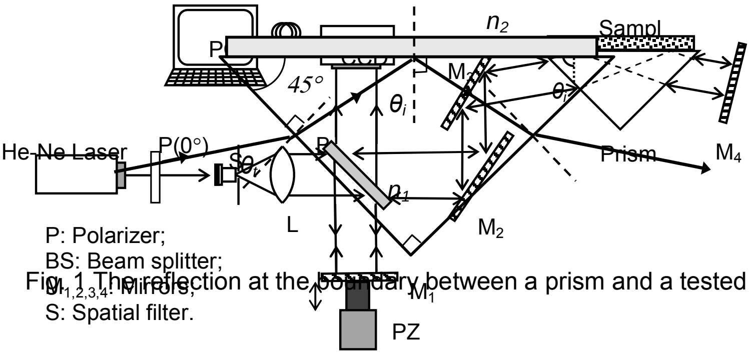

Fig. 1 The reflection at the boundary between a prism and a tested medium.

Fig. 2 Schematic diagram for measuring the two-dimensional distribution of refractive index of a material.

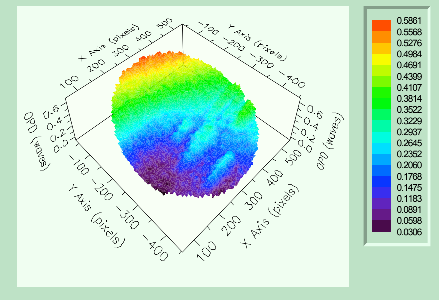

Fig. 3 The two-dimensional distribution of the phase variation of the test

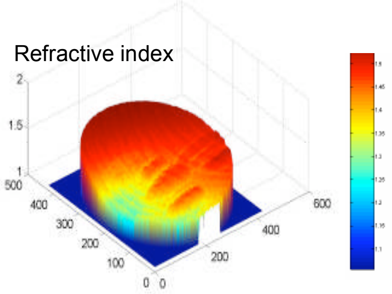

Fig. 4 The two-dimensional distribution