SPIE Newsroom

10.1117/2.1200806.1160

Electromagnetic response of thin metamaterial layers

Sergei Tretyakov and Constantin Simovski

Do the permittivity and permeability of thin metamaterial layers actually measure polarizations in the structure?

Much attention has been paid recently to metamaterials. These materials, which are designed to have advantageous and unusual electromagnetic properties, consist of artificial structural elements called inclusions. The hope is that ultimately metamaterials will enable a new range of optical applications. Yet, progress in the area has been limited by fundamental difficulties in describing the electromagnetic behavior arising from the materials' unusual features. Normally, researchers describe electromagnetic properties in terms of familiar material parameters such as permittivity and permeability (how strongly a material is polarized in electric and magnetic fields, respectively). They also have recourse to a powerful and very useful tool known as homogenization, which replaces the complex multiparticle structure of an electromagnetic material with a piece of effectively continuous matter.

With metamaterials, however, the distances between particles may be comparable to the wavelength of the light passing through them, whereupon permittivity and permeability lose their meaning. Moreover, samples may have only a few layers of inclusions across the layer thickness (optical metamaterial samples approach the extreme: they often have only one or two layers of particles). All of these peculiarities make it difficult to define material parameters and to interpret measured results in terms of familiar quantities. Unfortunately, the well-known strict and approximate numerical and analytical methods of quasistatic homogenization cannot be directly applied to metamaterials. The modern literature abounds in papers on this topic, but the results are contradictory, and often not reliable or even physically sound.[1–3]

This note reports our recent work on effective parameter modeling of thin metamaterial layers. Here, 'effective' refers to the use of a single parameter to describe the properties of a very complicated system. 'Thin' means that the layer has only a few inclusions across its thickness. Due to the resonant response of the structure, however, the phase shift of electromagnetic waves passing the layer may be significant, so the effective 'electrical thickness' is not necessarily small. One of the main challenges in studies of metamaterials is to design optical materials with left-handed behaviors (having negative permittivity and permeability). One starts by designing and manufacturing a single layer of nanosized complex-shaped particles (like the famous split rings) or a dual-layer structure (like the dual-bar structure). Does the structure really work as a material with negative parameters? And how can we measure them?

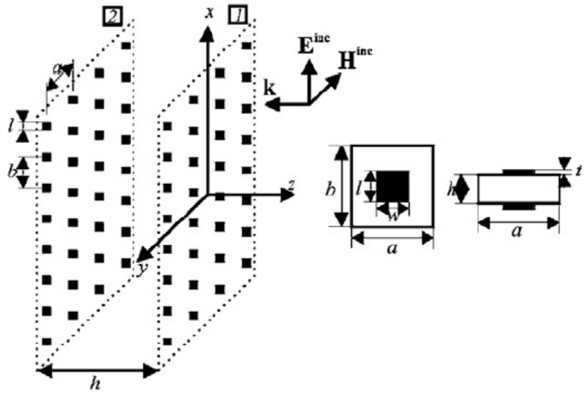

Figure 1. Geometry of a typical metamaterial structure with two layers of small metal inclusions. k: Wave vector of incident wave. E^{inc}: Electric field of incident wave. H^{inc}: Magnetic field of incident wave. Remaining letters are dimensions.

Usually one measures plane-wave reflection and transmission coefficients for the manufactured (or numerically simulated) layer and assumes that it should behave as a homogeneous material layer. Equating the measured or simulated reflection and transmission coefficients with the values for a homogeneous slab of the same thickness, the effective permittivity and permeability can easily be solved. This procedure is called S-parameter extraction. The questions are, What do these parameters actually measure? and How can they be used to predict behavior of the same structure in different environments? For instance, if we position

Continued on next page

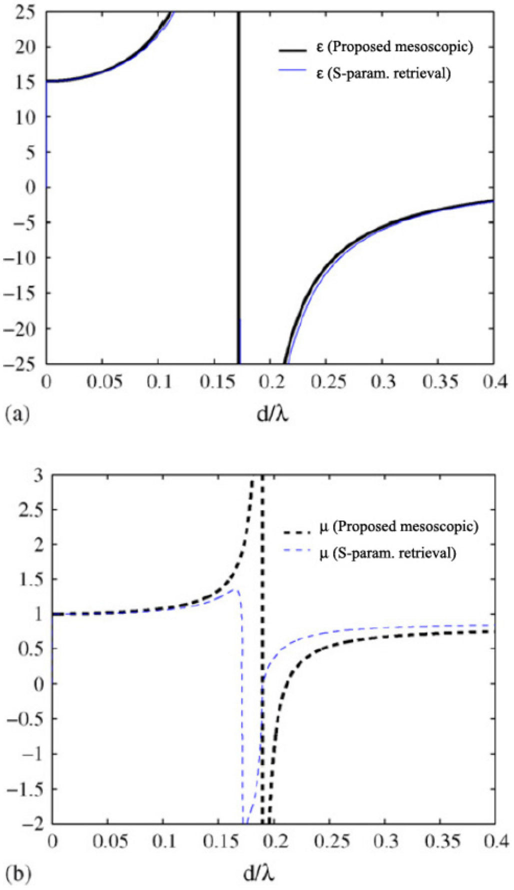

Figure 2. *Comparison between the mesoscopic effective material parameters obtained using calculated induced polarizations and the S-parameter retrieval method. (a) ε : Permittivity. (b) μ : Permeability. d: Distance between particles (period). λ: Wavelength.*

the same layer on a substrate with a different permittivity and repeat the exercise, do we get the same effective parameters? Or, if we illuminate the same layer from a different direction, and do the job again, do we get the same results? For a slab of a 'usual' material the answer is obviously yes. For the metamaterial layers the answer is, unfortunately, no.

The most exciting question is whether these material parameters extracted from the S-parameters of a composite slab actually measure the strength of electric and magnetic polarizations. In other words, if the extracted effective permeability is large, can we conclude that the layer has strong magnetic polarization? We have attempted to answer this question by detailed modeling of a simple structure, which is basically a single or double array of metal strips that can be continuous or broken into pieces (see an example geometry in Figure 1). In this simple case the currents induced on the inclusions can be analytically solved. Our idea is to explicitly calculate the induced electric and magnetic dipole moment densities and find ratios of induced dipole moment densities and volume-averaged fields inside the layer. This way we can find the layer parameters (we call them mesoscopic permittivity and permeability), which indeed measure the induced polarizations. Next, we can also extract the effective parameters from the calculated reflection and transmission coefficients. Figure 2 shows the results of comparison of effective parameters calculated from the actual induced polarizations as well as from the reflection and transmission coefficients. They are different! In the frequency range where the S-parameter extraction reveals that the medium has negative permeability, the actual magnetic polarization corresponds to a large positive permeability. Moreover, the frequency behavior of the S-parameter-based permeability is nonphysical. In this case we have neglected losses, which means that the permeability can only grow with the frequency (this follows from the causality principle). On the other hand, if we try to predict the reflection and transmission coefficients from the mesoscopic parameters, we do not get it exactly correct near the resonant frequency of the structure.

The conclusion is that neither of these two sets of effective parameters is perfect. One exactly restores the reflection and transmission coefficients but fails to measure the polarization properties of the layer. The other correctly describes the induced polarizations but is not accurate in predictions of the reflection and transmission coefficients. The reason for these difficulties is simple: the electromagnetic properties of considered thin metamaterial layers cannot be sufficiently described by two effective parameters. Their properties are more complicated. Further work is needed to develop alternative models, such as the grid impedance model used to describe sheets with negligible thickness.

Author Information

Sergei Tretyakov and Constantin Simovski
Helsinki University of Technology
Espoo, Finland

References

1. P. Ikonen, E. Saenz, R. Gonzalo, C. Simovski, and S. Tretyakov, *Mesoscopic effective material parameters for thin layers modeled as single and double grids of interacting loaded wires*, **Metamaterials 1** (2), pp. 89–105, 2007.
2. E. Saenz, P. Ikonen, R. Gonzalo, and S. Tretyakov, *On the definition of effective permittivity and permeability for thin composite layers*, **J. Appl. Phys. 101**, p. 114910, 2007.
3. C. R. Simovski and S. A. Tretyakov, *Local constitutive parameters of metamaterials from an effective-medium perspective*, **Phys. Rev. B 75**, p. 195111, 2007.