Probability and performance of vegetation indices

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A probabilistic approach should help assess the efficiency of various vegetation indices, used in mapping land cover.

A vegetation index (VI) is expressed as the ratio of reflectance of an object at the near infrared \((x)\) to its reflectance at the visible red \((y)\) band. It can also be given as a function of this ratio or the ratio of the linear combinations of \(x\) and \(y\). VIs are used in mapping vegetation cover with satellite imagery, with high and low values reflecting, respectively, density and sparsity. Numerous VI indices have been developed for various purposes. Most common is the normalized differences vegetation index (NDVI), defined as follows:

\[
\begin{align*}
    a &= \frac{x - y}{x + y}
\end{align*}
\]  

The assorted VIs are based mainly on empirical criteria of response over vegetation types, soils, and geological targets of interest.\(^1\)\(^-\)\(^3\) Data from remote sensing are compared with ground data, and various statistical parameters are computed in order to test reliability in mapping land cover and estimating biophysical parameters of interest. In some cases, VI sensitivity is computed with the aid of mathematical models that associate vegetation cover with its reflectance at different bands.\(^4\)\(^,\)\(^5\)

Recently, with the aid of probability theory, we developed an alternative methodology for assessing VI efficiency.\(^6\)\(^-\)\(^9\) Using this approach, quantitative evaluation of a VI is primarily based on values of the standard deviation (STDV) and the signal-to-noise ratio (SNR) of the images. An image of a VI with a large STDV and high SNR may be expected to offer good contrast and limited noise, enabling targets of interest to be expressed clearly.

In order to calculate the statistical parameters of a VI, we introduced the bi-dimensional distribution \(f(x, y)\), which describes the reflectance distribution at \(x\) and \(y\) bands, assuming a weak correlation between them.\(^6\)

\[
f(x, y) = 4a_x a_y e^{-a_x x^2 - a_y y^2}
\]  

where \(a_x\) and \(a_y\) are parameters that are inversely proportional to the square STDV of the \(x\) band and \(y\) band, respectively. Taking into account probabilistic theorems, it can be proved\(^6\) that the distribution \(g(u)\) of the NDVI values is given by:

\[
g(u) = \frac{4 \lambda (1 - u^2)}{\lambda (1 + u)^2 + (1 - u)^2)^2}
\]  

where \(\lambda\) is equal to \(a_x / a_y\).

It is possible to calculate numerically the STDV of \(g(u)\). Figure 1 presents its variation against \(\lambda\), which can be observed to have a maximum value of 1. This means that when STDVs of the near infrared and red bands are equal, that of the resulting NDVI image is maximum and tonality contrast is strongest. On the other hand, when \(\lambda\) is different from unity, the Modified NDVI (MNDVI) may help in producing an image with a high STDV. The MNDVI is defined as:

\[
\begin{align*}
    u &= \frac{c\lambda - y}{c\lambda + y}
\end{align*}
\]  

Using the value \(c\) equal to \(\sqrt{\lambda}\), the MNDVI image obtains its maximum STDV and tonality contrast.

The SNR ratio of an image may be defined\(^10\):

\[
\begin{align*}
    SNR &= \frac{\sigma}{\sigma_u}
\end{align*}
\]  

where \(\sigma_u\) is the STDV for the NDVI value of a certain pixel, which depends on the STDVs of \(x\) and \(y\) values of this pixel.
Also, $\sigma$ is the STDV of the tonality distribution $g(u)$. It can be proved that the SNR of the NDVI is given by:

$$\text{SNR} = \frac{\sigma_y (r + 1)}{2\sigma_n \sqrt{r^2 + 1}}$$

(6)

where $\sigma_n$ is the standard deviation of $x$ and $y$ values. Here, $r$ is equal to $x/y$. Figure 2 presents the variation of the SNR against $r$. It can be observed that the SNR increases (i.e., improves) with $r$. This approach has also been applied for other VIs.\textsuperscript{7-9}

In sum, our probabilistic approach may be useful both in assessing efficiencies of existing VIs and in introducing new ones with improved statistical properties. Thus far we have assumed the spectral bands have a weak or null correlation. We plan to study the effect of the correlation coefficient on VI performance in order to improve reliability.

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